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The influence of non-homogeneity on the frequency–amplitude characteristics of laminated orthotropic truncated conical shell

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ABSTRACT

In this study, the non-linear vibration of laminated non-homogenous orthotropic truncated conical shell is investigated. It is assumed that the Young's moduli, shear modulus and density of the layers of the shell vary exponentially through the thickness direction. The basic equations of laminated non-homogenous orthotropic truncated conical shells are derived using the large deformation theory with von Karman–Donnell-type of kinematic non-linearity. The non-linear basic equations are reduced to the non-linear differential equation depending on the time using the superposition principle and Galerkin method. This equation is solved using semi-inverse method and is found the frequency–amplitude relationship. Finally, carrying out some computations, the effects of non-homogeneity, number and ordering of layers, and conical shell characteristics on frequency–amplitude characteristics have been studied.

1. Introduction

Conical shells are one of the necessary structural components and commonly found in a variety of engineering applications, such as the military, aerospace, turbo-machinery and ship building industries; these great applications impel researchers to study the dynamic performance of conical shells. Due to the sensitive nature of the scope of conical shells, the accurate evaluation of their non-linear vibration characteristics under different working conditions becomes essential. The first attempts to solve the problem of non-linear vibration of single-layer isotropic conical shells was done by Ueda [1] and so far researches on this topic continues [2–4]. A comprehensive review on the non-linear vibrations of shells may be found in book of Amabili [5].

The developments of material science brought composite materials out, the demands of high stiffness-to-weight and high strength-to-weight for structures made them applicable in engineering in the form of laminated composite shells and the tendency is increasing. These developments in the material science have increased the interest of scientists on the non-linear vibration problems of conical shells made of laminated composites. The corresponding researches for the non-linear vibrations of homogeneous laminated composite shells were done by Liu and Li [6]; Xu et al. [7]; Lakis et al. [8]; Civalek [9,10]; Naidu and Sinha [11]; Shen [12] Viola et al. [13]. Important contributions to the study of the linear and non-linear vibrations of laminated shells have been carried out in the last three decades. A review of many of these works has been given by Reddy [14] and Qatu [15].

In many practical applications, shell type structural elements have to operate under radiation, moisture and elevated temperature environment, which cause non-homogeneity in the material. Understanding the effects of moisture and temperature in the behavior of composite structures is necessary in aerospace, military and civilian applications because many composite structural parts and military installation encounter high and low temperature variations in the presence of moisture [16]. A composite material may gain about three percent in weight due to absorption of moisture when exposed to a humid environment [17]. However, structural components are often non-homogeneous, because of design, manufacturing process, production techniques, surface and thermal polishing processes in the underlying material. Thus, the mechanical properties of materials change from point to point as random, piecewise continuous or continuous functions of space coordinates [18]. The use of such shells as structural elements in various technological situations demands that the non-homogeneity of the materials should be taken into account for the analyses of vibration problems. Various models representing the behavior of the non-homogeneous materials have been proposed in the literature and given in Refs. [19–21]. In these references various models such as linear, quadratic, exponential etc. for the Young moduli, shear modulus and density of the plate and shell materials have been considered.

A review of the literature shows that few studies have been carried out to investigate the vibration of non-homogeneous laminated shells. Mecitoglu [22] studied the dynamic behavior of in-homogeneous composite laminated truncated conical thin shell.

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Zenkour and Fares [23] studied the bending, buckling and free vibration of non-homogeneous composite laminated cylindrical shells using a refined first-order theory and Hamilton–Reissner's mixed variational principle. Wu and Lee [24] investigated the free vibration analysis of laminated conical shells with variable stiffness is presented using the method of differential quadrature (DQ). The stiffness coefficients are assumed to be functions of the circumferential coordinate that may be more close to the realistic applications. Aksogan et al. [25] studied linear free vibration analysis of cross-ply laminated non-homogeneous composite truncated conical shells. The material properties are assumed to vary continuously through the thickness direction of the layers, according to a power law distribution. Batra and Jin [26] investigated natural frequencies of a functionally graded anisotropic rectangular plate. Tripathi et al. [27] presented the sensitivity of randomness in material parameters on linear free vibration response of conical shells. A finite element method is successfully combined with first-order perturbation technique to obtain the response statistics of the structure. Wang and Sudak [28] studied three-dimensional analysis of multi-layered functionally graded anisotropic cylindrical panel under thermomechanical loading. Sofiyev and Karaca [29] examined the linear free vibration and buckling of laminated homogeneous and non-homogeneous orthotropic truncated conical shells under lateral and hydrostatic pressures, which are the Young's moduli and density of the layers, vary continuously in the thickness direction.

There are not many solutions for non-homogeneous laminated conical shells based on the large deformation theory because of the considerable mathematical difficulties in solving the governing differential equations and numerical analyses [30].

In a recent survey of the literature the authors have found no work dealing with non-linear vibration of non-homogeneous laminated conical shells. Due to complicated effects such as strong influences of non-linearity, non-homogeneity, anisotropy and lamination, and so on, the dynamic behavior of such advanced structural elements is considerably more complicated than that for the corresponding homogeneous isotropic cases. Hence, accurate prediction of their dynamic response often requires analyses that are based on the large deformation theory considerations rather than small deformation theory. Therefore, it is very important to develop an accurate, reliable analysis towards the understanding of the non-linear vibration characteristics of the laminated non-homogenous orthotropic conical shells. In this paper, the non-linear vibration behaviors of laminated non-homogeneous orthotropic conical shells are investigated by using non-linear Donnell shell theory. The non-linear basic equations are reduced to the non-linear differential equation depending on the time using the superposition principle and Galerkin method. This equation is solved using the Ritz and semi-inverse methods and is found the frequency–amplitude relationship. Numerical results show various effects of the number and ordering of layers, non-homogeneity, orthotropy and conical shell characteristics on the frequency–amplitude characteristics.

2. Basic equations

Consider a thin laminated circular truncated conical shell as shown in Fig. 1, having N layers of equal thickness made of non-homogeneous orthotropic materials. The layers are assumed to be perfectly bonded at their interfaces. Here h is the total thickness, L is the length and γ is the semi-vertex angle of the conical shell. R_1 and R_2 indicate the radii of the cone at its small and large ends, respectively. S_1 is the distance from the vertex to the small base. A set of curvilinear coordinates (ζ, θ, S) is located on the reference surface. The reference surface $\zeta = 0$ is located at a layer interface for even values of N, whereas, for odd values of N the reference surface is located at the center of the middle layer. The ζ-axis is always normal to the moving S-axis, lies in the plane generated by the S-axis and the axis of the cone, and points inwards. The θ-axis is in the direction perpendicular to the $S-\zeta$ plane. Ψ be the stress function for the stress resultants defined by $N_S = \Psi_{,\theta_1\theta_1}/S^2 + \Psi_{,S}/S,\ N_\theta = \Psi_{,SS},\ N_S = -\Psi_{,S\theta_1}/S + \Psi_{,\theta_1}/S^2$, where $\theta_1 = \theta\sin\gamma$ and a comma denotes partial differentiation with respect to the corresponding coordinates. The axes of orthotropy in all layers are parallel to the S and θ axes.

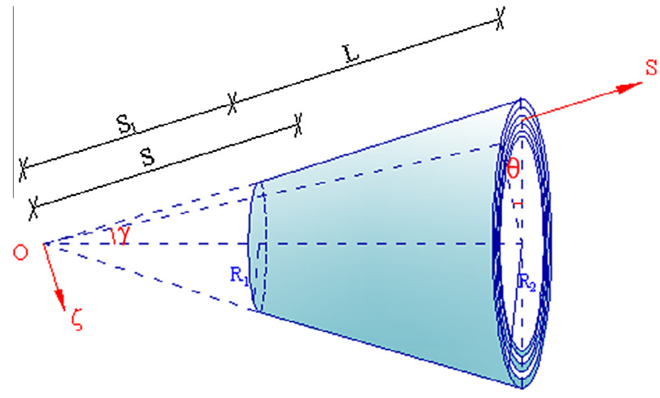

Fig. 1. Geometry of the laminated truncated conical shell and notations.

The Young's moduli, shear modulus and density of the layers are defined as continuous functions of the thickness coordinate ζ as $\left[E_S^{(k+1)}(\bar{\zeta}), E_\theta^{(k+1)}(\bar{\zeta}), G^{(k+1)}(\bar{\zeta})\right] = \bar{\varphi}_1^{(k+1)}(\bar{\zeta})\left[E_{0S}^{(k+1)}, E_{0\theta}^{(k+1)}, G_0^{(k+1)}\right]$, $\rho^{(k+1)}(\bar{\zeta}) = \rho_0^{(k+1)}\bar{\varphi}_2^{(k+1)}(\bar{\zeta})$, where $-0.5 + k/N \leqslant \bar{\zeta} \leqslant -0.5 + (k+1)/N,\ \bar{\zeta} = \zeta/h,\ k = 0, 1, \ldots, N-1,\ E_{0S}^{(k+1)}$ and $E_{0\theta}^{(k+1)}$ are Young's moduli of homogeneous materials in the layer, $(k+1)$, along the S and θ directions, respectively, $G_0^{(k+1)}$ is the shear modulus of homogeneous materials on the plane of the layer, $(k+1)$ and $\rho_0^{(k+1)}$ is the density of homogeneous materials in the layer, $(k+1)$. Additionally $\bar{\varphi}_i^{(k+1)}(\bar{\zeta}) = 1 + \mu\varphi_i^{(k+1)}(\bar{\zeta})(i = 1, 2),\ k = 0, 1, \ldots, N-1$, where $\varphi_i^{(k+1)}(\bar{\zeta})$, $(i = 1,2)$ are continuous functions giving the variations of the Young's moduli, shear modulus and density in the layers, satisfying the condition $\left|\varphi_i^{(k+1)}(\bar{\zeta})\right| \leqslant 1$; μ is a variation coefficient of the Young's moduli and density, satisfying $0 \leqslant \mu \leqslant 1$ [29].

The constitutive equations for the non-homogeneous orthotropic layers are given as follows [29,30]:

$$\begin{bmatrix} \sigma_S^{(k+1)} \\ \sigma_\theta^{(k+1)} \\ \sigma_{S\theta}^{(k+1)} \end{bmatrix} = \begin{bmatrix} Q_{11}^{(k+1)}(\bar{\zeta}) & Q_{12}^{(k+1)}(\bar{\zeta}) & 0 \\ Q_{12}^{(k+1)}(\bar{\zeta}) & Q_{22}^{(k+1)}(\bar{\zeta}) & 0 \\ 0 & 0 & Q_{66}^{(k+1)}(\bar{\zeta}) \end{bmatrix} \begin{bmatrix} e_S - \zeta\frac{\partial^2 w}{\partial S^2} \\ e_\theta - \zeta\left(\frac{1}{S^2}\frac{\partial^2 w}{\partial\theta_1^2} + \frac{1}{S}\frac{\partial w}{\partial S}\right) \\ e_{S\theta} - \zeta\left(\frac{1}{S}\frac{\partial^2 w}{\partial S\partial\theta_1} - \frac{1}{S^2}\frac{\partial w}{\partial\theta_1}\right) \end{bmatrix} \quad (1)$$

where $\sigma_S^{(k+1)},\ \sigma_\theta^{(k+1)},\ \sigma_{S\theta}^{(k+1)}$ are the stresses in the layers, $e_S,\ e_\theta,\ e_{S\theta}$ are the strains on the reference surface and the quantities $Q_{ij}^{(k+1)}(\bar{\zeta})$, $i, j = 1, 2, 6$, for non-homogeneous orthotropic lamina are

$$Q_{11}^{(k)}(\bar{\zeta}) = \frac{E_{0S}^{(k+1)}\bar{\varphi}_1^{(k+1)}(\bar{\zeta})}{1 - \nu_{S\theta}^{(k+1)}\nu_{\theta S}^{(k+1)}},\quad Q_{12}^{(k+1)}(\bar{\zeta}) = \nu_{\theta S}^{(k+1)}Q_{11}^{(k+1)}(\bar{\zeta}) = \nu_{S\theta}^{(k+1)}Q_{22}^{(k+1)}(\bar{\zeta}),$$
$$Q_{22}^{(k)}(\bar{\zeta}) = \frac{E_{0\theta}^{(k+1)}\bar{\varphi}_1^{(k+1)}(\bar{\zeta})}{1 - \nu_{S\theta}^{(k+1)}\nu_{\theta S}^{(k+1)}},\quad Q_{66}^{(k+1)}(\bar{\zeta}) = 2G_0^{(k+1)}\bar{\varphi}_1^{(k+1)}(\bar{\zeta}),\quad k = 0, 1, \ldots, N-1 \quad (2)$$

in which $\nu_{S\theta}^{(k+1)}$ and $\nu_{\theta S}^{(k+1)}$ are the Poisson's ratios of the layers, $(k+1)$, assumed to be constant.

Based on von Karman–Donnell-type non-linear strain–displacement relations, the strain components on the middle plane of laminated truncated conical shells are expressed by [31]