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A layerwise theory for laminated composites in the framework of isogeometric analysis

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ABSTRACT

Through-thickness modeling of laminated composites using a displacement-based isogeometric layerwise theory is presented. Layerwise theories provide accurate predictions of the three-dimensional stress states that are of prime importance in structural design. This is in sharp contrast to the class of equivalent-single-layer theories that yield no or limited information of three-dimensional stress states. The key idea of layerwise theories is to distinguish and separate the functions of approximation employed in the in-plane and out-of-plane directions. The rationale behind this choice emanates from the underlying physics, due to the balance of linear momentum and continuity of traction, the function describing the transverse displacement field should be C^0-continuous at the interface between plies of different fiber angle orientation. The latter condition can be naturally facilitated through conscious use of isogeometric refinement schemes. Finally, a multiple model analysis is introduced. The aim is to demonstrate the use of the different models within predefined regions of a single laminate. The multiple model analysis concept is employed to simulate laminates with existing delaminations. The proposed models are verified considering laminated composite plates. The numerical results confirm the accuracy of the proposed models and shown that the isogeometric layerwise model outperforms its Lagrange polynomial-based counterpart.

1. Introduction

Fiber-reinforced composite structures are increasingly used in a wide range of industries. To date, various methods have been proposed for the analysis of composite laminates, see e.g. the classical references by Carrera [1] or Reddy [2]. Depending on the displacement and/or stress expansions through the laminate thickness, two main categories of theories can be distinguished: the equivalent-single-layer (ESL) and the layerwise (LW) theories. Equivalent-single-layer theories can be further classified into classical shell formulations and homogenization-based approaches. Classical shell formulations reduce a three-dimensional continuum problem to a two-dimensional one by expanding the displacement field as a linear combination of predefined or known functions of the thickness co-ordinate and integrating the constitutive law through the thickness either analytically or numerically [3]. Alternatively, stiffness properties may be homogenized through the thickness of the laminate without reducing the geometric dimension of the problem. Although ESL theories may be adequate for describing the behavior of thin composite shells, they typically fail to capture (accurately) the complete three-dimensional stress field at the ply level in moderately thick, and thick laminates. This deficiency is primarily associated with the fact that transverse strain components are incorrectly assumed to be continuous across the interface of dissimilar materials, which entails non-physical local discontinuity of the transverse stresses.

In contrast to ESL theories, layerwise techniques assume separate displacement field expansions within each layer. Following equilibrium considerations, the transverse displacement component is defined to be C^0-continuous at ply interfaces and thereby yield a more accurate description of the complete stress state. In most displacement-based layerwise models [4–10], C^0-continuity of the displacement field across layer interfaces is imposed through constructing elaborate displacement functions or through adding constraint equations at layer interfaces. For instance, in reference [8] each layer is modeled as an independent plate, then the compatibility of displacement components at layer interfaces are imposed through the use of Legendre polynomials. Alternatively, one-dimensional through the thickness Lagrangian finite elements are used to approximate three components of the displacement field which automatically enforces C^0-continuity conditions at layer interfaces, see e.g. [11–13]. The latter approach results in a

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continuous in-plane and discontinuous transverse strain field, allowing for the possibility of continuous transverse stresses at the layer interfaces. Furthermore, compared to conventional 3-D finite element models, the layerwise model is computationally more efficient while retaining the same modeling capabilities [12].

It is interesting to note that equivalent-single-layer and layerwise theories may be successfully casted into a unified framework making them more practical to use, see e.g. [14,15]. The non-evident choice between various ESL and LW models is primarily driven by the geometry of the part, the material properties, or even the stacking sequence of the laminate. Equivalent-single-layer models tend to require less modeling effort from the designer and in most cases offer reduced computational times. Usually, these benefits come at the expense of incomplete or even inaccurate results, except for the model developed in [15]. In contrast, layerwise models provide more accuracy and are computationally more intense. Consequently, combining these two types of models should allow us to solve structural problems using a reasonable amount of computational resources at a reduced cost. This method is often denoted *multiple model analysis* [2], and it is a general case of the commonly used *simultaneous global–local strategy* [16]. Here global refers to the entire structure modeled using an ESL theory except for a set of critical subdomains described by a layerwise model. In multiple model analysis, the main difficulty often lies in the coupling of incompatible meshes and/or different mathematical models, i.e. to maintain the kinematic compatibility and continuity of traction at the boundaries of adjacent regions. In order to address this difficulty, Whitcomb and Woo have [17,18] developed an iterative method to establish the force equilibrium conditions at global–local boundaries. In some other works, multipoint constraints [19] or transition elements [20] are used to connect different mathematical models. Reddy et al. [21,16] have proposed a more robust global–local analysis method which is called the variable displacement field method, so that in a given region of domain, all appropriate part of the displacement field can be invoked. Displacement continuity is enforced between different types of regions.

Isogeometric analysis, introduced by Hughes et al. [22], is a novel concept in computational mechanics aimed at unifying computer aided design (CAD) and finite element analysis (FEA). In contrast to classical FEA where the geometry and the unknown solution field(s) are approximated with Lagrange polynomials, isogeometric analysis (IA) employs the basis functions used to describe the geometry to approximate the physical response in an isoparametric sense. The accuracy and efficiency of the isogeometric paradigm has been validated by a number of researchers, see e.g. [23–26].

To date only few references can be found that employ the isogeometric concept to investigate composite laminates [27–32]. Most of these papers are based on classical lamination theory (CLT) or first-order shear deformation theory both belonging to the class of ESL methods. Recently, Thai [33] proposed an isogeometric layerwise theory in which a first-order shear deformation theory is used in each layer, and the displacement continuity at layer interfaces is imposed.

In this paper, we focus on through-the-thickness modeling of laminated composites in the framework of higher order and higher continuity NURBS. The superiority of isogeometric paradigm in the modeling of composite laminates is demonstrated through several numerical examples in the state of cylindrical bending. In particular, we exploit the unique *k*-refinement capabilities of isogeometric analysis to reveal the method's potential for models based on the proposed layerwise theory. Next, an isogeometric based multiple model method is presented and an error study using several different parameters is carried out. Finally, we investigate a multiple model that includes delamination in a predefined region.

The remaining part of this paper is built up as follows. Fundamentals of non-uniform rational B-splines and isogeometric analysis are presented in Section 2. The details and verification of isogeometric layerwise model, multiple model method and the delamination modeling are presented in Sections 3 and 4, respectively. Finally, conclusions are given in Section 5.

2. NURBS and isogeometric analysis

2.1. Non-uniform rational B-splines

An open knot vector, Ξ, is a non-decreasing sequence of real numbers defined as:

$$\Xi = \{\xi_1 = \cdots = \xi_{p+1} = 0, \xi_{p+2}, \ldots, \xi_n, \xi_{n+1} = \cdots \xi_{n+p+1} = 1\}, \tag{1}$$

where ξ_i, with $i = 1, 2, \ldots, n + p + 1$, p, and n denote a knot, the degree, and number of the basis functions, respectively. In essence, the parametric space, defined by the knot vector, is subdivided by the knots into knot spans, i.e. $[\xi_j, \xi_{j+1})$. Furthermore, repeating starting and end knots in Eq. (1) ensure endpoint interpolation of the defined geometry.

Given the knot vector, one can introduce B-spline functions using the Cox-de Boor recursion formula [34]. The recursion starts with degree $p = 0$ basis functions which are defined as:

$$N_{i,p=0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leqslant \xi < \xi_{i+1}, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

where $N_{i,\,p=0}$ is the ith piecewise constant basis function and $\xi \in [0, 1]$ is the variable of parametrization. Basis functions of degree $p > 0$ are constructed as:

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi). \tag{3}$$

A pth degree NURBS curve, $\mathbf{c}(\xi)$, is furnished as [35]

$$\mathbf{c}(\xi) = \sum_{i=1}^{n} \boldsymbol{P}_i R_{i,p}(\xi), \tag{4}$$

where $\boldsymbol{P}_i$ is a control point and $R_{i,\,p}(\xi)$ designate a rational basis function defined as:

$$R_{i,p} = \frac{N_{i,p}(\xi) w_i}{\sum_{j=1}^{n} N_{j,p}(\xi) w_j}. \tag{5}$$

The symbol w_i in Eq. (5) denotes the weight associated with the ith control point.

Let us now assume that the two (distinct) knot vectors Ξ_1 and Ξ_2 are given, following Eqs. (1) and (2) one may define n_1 and n_2 basis functions of degree p_1 and p_2, respectively. Making use of the tensor product scheme, the NURBS surface, $\boldsymbol{s}(\xi, \eta)$, can be written as:

$$\boldsymbol{s}(\xi_1, \xi_2) = \sum_{I=1}^{K} \boldsymbol{P}_I R_I(\xi_1, \xi_2) = \boldsymbol{P}^T \boldsymbol{r}, \tag{6}$$

where $K = n_1 \cdot n_2$, $\boldsymbol{P}_I$, and $R_I(\xi_1, \xi_2)$ denotes the number of rational basis functions, the control points comprised in the control net, and the bivariate rational basis functions. The symbols $\boldsymbol{P} \in \mathbb{R}^{K \times d}$, with $d = 2, 3$ denoting the spatial dimensions, and $\boldsymbol{r} \in \mathbb{R}^K$ in Eq. (6) represent the control point matrix and the interpolation vector respectively. For notational brevity, we introduced a single index notation in Eq. (6). For instance, the bivariate NURBS basis function, which is constructed by the tensor product of one dimensional shape functions, is defined as:

$$R_{I(i_1,i_2)} : R^{p_1,p_2}_{i_1,i_2}(\xi_1, \xi_2) = \frac{N_{i_1,p_1}(\xi_1) N_{i_2,p_2}(\xi_2) w_{i_1,i_2}}{\sum_{j_1=1}^{n_1} \sum_{j_2=1}^{n_2} N_{j_1,p_1}(\xi_1) N_{j_2,p_2}(\xi_2) w_{j_1,j_2}}, \tag{7}$$