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# Buckling analysis of variable angle tow, variable thickness panels with transverse shear effects ${}^{\star}$

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#### ABSTRACT

The manufacture of advanced composite panels with variable fibre angles can lead to laminates with a flat profile on one side and a smooth, curved profile on the other. When modelling these laminates in two-dimensional form the flat plate assumptions may no longer accurately capture the structural behaviour. In this paper the buckling behaviour of laminates with one-dimensional fibre variations and symmetric stacking sequences is investigated. The assumptions of modelling the three-dimensional profile as a flat plate or a cylindrical panel are assessed, taking into account the effects of transverse shear deformation. The governing differential equations are solved in the strong form using the Differential Quadrature method and validated by 2D finite element models. The validity of the two modelling approaches is assessed by comparing the solutions to a 3D finite element model capturing the actual shape of the laminate. It is suggested that the buckling event of these variable angle tow, variable thickness laminates is characterised more accurately by "shell-like" than by "plate-like" behaviour. The idea of investigating the effects of two-dimensional fibre orientations with their associated doubly curved topologies is proposed. © 2013 The Authors. Published by Elsevier Ltd. All rights reserved.

## 1. Introduction

The idea of tailoring the structural performance of composite laminates by spatially varying the point-wise fibre orientations over the planform has been explored since the early 1990s. For example, the work by Hyer and Lee [1] and Hyer and Charette [2] showed that such variable angle tow (VAT) laminates can improve the stress concentration around holes by arranging the fibres in the direction of critical load paths. In recent years the use of fibre re-inforced composites in primary aircraft structures has led to increased interest in VAT technology. Numerous works have shown that tailoring the in-plane stiffness of a plate allows pre-buckling stresses to be re-distributed to supported regions and thereby improve the buckling behaviour [3–7]. In this manner VAT technology has been shown to improve the buckling performance of a composite fuselage window section by 12% compared to an equivalent straight-fibre laminate [8] and alleviate the pressure pillowing of fuselage sections [9]. Recent results by Wu et al. [10] show that VAT plates with linear fibre variations can be designed to exhibit smaller stiffness reductions in the post-buckling regime than their straight-fibre counterparts. Besides, the optimum fibre orientations for increasing the

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Currently, the major technology for manufacturing VAT laminates is the automated fibre placement (AFP) technique developed since the 1980s. AFP uses a robotic fibre placement head that deposits multiple pre-impregnated tows of "slit-tape" allowing cutting, clamping and restarting of every single tow. While the robotic head follows a specific fibre path, tows are heated shortly before deposition and then compacted onto the substrate using a special roller. Due to the high fidelity of current robot technology AFP machines can provide high productivity and handle complex geometries [12]. However, in AFP steering is accomplished by bending the tows of fibres in-plane, which leads to local fibre buckling on the inside radii of the curved tow and thus limits the steering radius of curvature [13]. Furthermore, if individual tows are placed next to each other by shifting the reference path along a specific direction, tow gaps and overlaps are inevitably required to cover the whole surface. To overcome the drawbacks of AFP the continuous tow shearing technique (CTS) was developed which uses shear deformation to steer fibres at the point of application [14]. This technique not only allows much tighter radii of curvature but tow gaps and overlaps are also avoided by tessalating tows on the substrate [14]. One feature of CTS is that in order to maintain the volume fraction of fibre the thickness of a tow inherently increases as it is sheared. The relation between unsheared tow thickness  $t_0$  and sheared tow thickness  $t_{\theta}$  is

$$t_{\theta} = \frac{t_0}{\cos \theta} \tag{1}$$





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buckling load are similar to those for minimising the transverse displacement in the post-buckling regime [11].

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where  $\theta$  is the shearing angle of the tow. Consequently, the thickness of a ply may locally increase by a factor of 4 if the fibre tow is sheared through an angle of 75°. As the laminate is cured on a tool plate one side of the laminate remains flat while the other resembles a curved panel as depicted in Fig. 1. The effects of this one-sided thickness profile in terms of local three-dimensional stress fields or buckling behaviour is currently unexplored.

Furthermore, since composite laminates are more affected by transverse shear effects than isotropic materials [15,16] these local areas of increased thickness make predictions of the buckling behaviour using Classical Laminate Analysis (CLA) [17] overly conservative. Buckling optimisation algorithms using the Finite Element method (FEM) can become computationally expensive, and do not readily shed physical insight into the fundamental mechanisms in which these thickness variations could influence the buckling behaviour. In fact, there is an inherent need in the aerospace industry for rapid vet accurate analysis tools to aid in preliminary design studies. For this reason a reduced 2D equivalent single-layer formulation for the flexural behaviour of VAT plates incorporating transverse shear effects was developed [18]. The model accurately predicts global bending and buckling behaviours up to thickness to length ratios of 1:10 while restricting the number of unknowns to only four variables to enhance computational efficiency. It was shown that CLA predictions result in discrepancies in excess of 15% for thickness to length ratios of 1:20. Thus, considering the extent of thickness variation possible using CTS, transverse shear effects need to be incorporated if the spatial fibre distribution is to be optimised accurately for buckling and postbuckling behaviour.

In this work, the elastic stability of VAT panels with linear fibre variations manufactured using the CTS technique is analysed using two different approaches. The aim is to compress the three-dimensional shape of the laminate as seen in Fig. 1 onto an equivalent single layer. However, due to the differences in topology between the top and bottom surfaces the shape of the chosen reference



**Fig. 1.** CTS manufactured  $0\langle 0|-70\rangle$  layer with reference path shifted in the Y-direction showing fibre orientations and thickness variations. The 3D structure can be compressed onto an equivalent single layer resembling either a cylindrical shell (Curve 1) or a non-symmetric flat plate (Curve 2).

plane is not immediately obvious. First, the VAT panel is assumed to behave as a flat, non-symmetric plate and the aforementioned 2D equivalent single-layer formulation is applied to solve the buckling problem. Second, and the major novelty of this work, is to assume that the smooth thickness variations cause the structure to behave more like a curved panel, in which case the equilibrium equations of cylindrical shells must be used.

This premise creates the opportunity for interesting optimisation studies since non-prismatic fibre variations cause the curved side of the VAT panel to take on more complex, doubly-curved shapes. In this case the need for using the most general set of shell kinematics may reveal different and perhaps enhanced buckling solutions compared to previous surveys based on flat plates [5,6,8,9]. Furthermore, the relatively shallow curvature produced by the thickness variations suggests that the unstable nature of shell post-buckling paths may be contained to a large extent. Thus, the idea of a structure with higher "shell-like" buckling loads in conjunction with stable "plate-like" characteristics in the postbuckling regime is suggested. For the purposes of this paper we are solely interested in solving the buckling problem of VAT panels with prismatic fibre variations that cause cylindrical shapes of the reference surface. Buckling and post-buckling studies of more general VAT panels as discussed above will be the topic of future work.

A large number of studies regarding the solution of von Kármán's non-linear differential equations for the postbuckling behaviour of plates using either energy methods [19-22] or Fourier-series expansions [23,24] were published in the early to mid-part of the 20th century. In the present work, the perturbation technique demonstrated by Stein [25] for rectangular isotropic plates and later developed by Chandra and Raju [26] for symmetrically laminated, orthotropic plates is applied. In this technique the non-linear differential equations are reduced to an infinite set of linear differential equations by expanding the unknown functional fields in a power series of an arbitrary perturbation parameter about the point of buckling. The solution of the first equation of the infinite set provides the pre-buckling stresses in the structure, while the eigenvalues and eigenmodes of the second equation yield the buckling loads and modes. Finally, succeeding equations beyond this point relate to large-deflection postbuckling solutions.

The rest of the paper is laid out as follows. Section 2 briefly describes the theoretical background of the aforementioned 2D equivalent single-layer model and outlines the governing equations needed to model the laminate as a shell. Furthermore, the perturbation technique for solving the ensuing non-linear shell equations is presented in more detail. In Section 3, the governing equations are solved in the strong form for various load and boundary conditions. The analytical results are validated by 2D FEM and 3D FEM techniques and the accuracy of the "plate-like" and "shell-like" modelling approaches discussed over a range of realistic fibre trajectories. Finally, conclusions are drawn in Section 4.

### 2. Theory

#### 2.1. Modelling of variable thickness cross-section

Linear fibre variations in one direction only (i.e prismatic variation) can conveniently be defined by the notation  $\theta = \phi \langle T_0 | T_1 \rangle$ , where  $\phi$  denotes the rotation of the fibre path with respect to the *x*-axis, while  $T_0$  and  $T_1$  are the fibre angles at the ply centre and a characteristic length *d* from the centre respectively [3]. To fill the planform the fibre trajectories are then shifted perpendicular to the steering direction  $\phi$ . A  $0\langle 0|-70\rangle$  VAT layer is drawn schematically in Fig. 1. Download English Version:

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