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AC electroosmotic flow of generalized Maxwell fluids in a rectangular microchannel

Yong-jun Jian *, Quan-sheng Liu, Lian-gui Yang

School of Mathematical Science, Inner Mongolia University, Hohhot 010021, China

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ABSTRACT

Many biofluids such as blood and DNA solutions are viscoelastic and exhibit extraordinary flow behaviors, not existing in Newtonian fluids. In the present investigation, analytical series solutions for the time periodic EOF flow of the generalized Maxwell fluids through a two-dimensional rectangular microchannel are found under the Debye–Hückel linear approximation. A linearized Poisson–Boltzmann equation governing the electrical double layer (EDL) field, together with the Cauchy momentum equation and the general Maxwell constitutive equation are included in the analysis. Numerical results are presented for the velocity profiles and volumetric flow rates in the rectangular microchannel for different parametric values that characterize this flow.

1. Introduction

Microfluidics is one of the most important research areas in micro-electro-mechanical-systems (MEMS) due to its potential applications as a tool for studying fundamental physical and biochemical processes and a platform for performing chemical and biological assays [1–3]. Most material surfaces acquire electric charges when in contact with a polar medium. These surface charges attract counter-ions dissolved in the contacting solution. As a result, the counter-ions gather in excess over the co-ions in the immediate vicinity of the charged surface, forming the Stern layer, a layer of typical thickness of one ionic diameter. Right next to the Stern layer, the diffuse layer is formed, which contains both co-ions and counter ions, and its ion density variation obeys the Boltzmann distribution [4]. The Stern layer and the diffuse layer constitute the so-called electrical double layer (EDL). The characteristic thickness of the EDL is commonly known as the Debye length. Under the influence of an electric field applied tangentially along the charged surface, the positive ions (cations) in the electrolyte migrate toward the cathode and the negative ions (anions) toward the anode. The migration of the mobile ions will carry the adjacent and bulk liquid phase by viscosity, resulting in an electroosmotic flow (EOF). Compared to conventional mechanical pumping modes, electroosmosis-based pumping has numerous advantages, including ease of fabrication and control, absence of mechanical parts, high reliability and no noise, etc.

Theoretical, numerical and experimental investigations of steady EOF of Newtonian fluids have been well studied in various micro-capillaries geometric domains such as slit parallel plate [5], cylindrical capillary [6,7], annulus [8,9], elliptical pore [10], rectangular [11–14], T-shape [15], semicircular [16] and sector [17] microchannel cross-sections.

Recently, periodical EOF attracts growing attention as an alternative mechanism of microfluidic transport because it produces electrokinetic instability in microchannel and enhances liquid mixing for low Reynolds number flows [18]. Many chemical processes in the human body, for example, the effects of cellular phones on human brains may be related to periodical EOF [19]. In geosciences, periodical EOF has been used to investigate the flow behavior in capillary and determine rock permeability in porous media [20]. Steady electroosmosis has a pluglike velocity profile across the channel. Quite differently, periodical electroosmosis has a wavelike velocity profile varying with time. Dutta and Beskok [21] were among the early researchers who analytically investigated the time periodic EOF between two parallel plates, illustrating interesting similarities to the Stokes second problem. Without assumption of the thin EDL thickness, Keh and Tseng [22] studied transient electrokinetic flow in one-dimensional capillaries. By using Green's function method, Kang et al. [23] obtained analytically transient EOF field in response to the application of time dependent electric field in cylindrical microchannels. With the aid of Fourier series and Laplace transforms, Campisi et al. [24] investigated transient and steady-state electrokinetic phenomena in infinitely extended rectangular charged microchannels. The expressions of flow velocity profiles, flow rates, streaming currents, and complex hydraulic and electrokinetic conductances were

* Corresponding author. Tel.: + 86 471 4992946x8313; fax: +86 471 4991650.
E-mail address: jianyongjun@yahoo.com.cn (Y.-j. Jian).

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obtained. A semi-analytical solution of periodical EOF in a rectangular microchannel was presented by Wang et al. [25]. They found that the velocity of periodical EOF strongly depends on the Reynolds number, the properties of EDL, and the applied electric field. Chakraborty and Ray [26] investigated the mass flow-rate control through time periodic EOF in circular microchannels. Chakraborty and Srivastava [27] extended the work of Qu and Li [28] about overlapping EDL for time periodic EOF. Recently, Jian et al. [29] derived an analytical solution of velocity distribution for time periodic EOF in a cylindrical microannulus. Two limiting cases, i.e., the time periodical EOF approximately in parallel plate microchannel and circular microtube are discussed in their work.

All the papers indicated above, and references therein, deal with Newtonian fluids. However, complex fluids like polymer solutions, colloids, and cell suspensions are also manipulated in microfluidic devices. These polymer solutions are made of long chain molecules and behave obviously non-Newtonian characteristics. The theoretical study of electroosmotic flows of non-Newtonian fluids is recent and has been mostly limited to simple fluid models due to the inherent analytical difficulties introduced by more complex constitutive equations. Das and Chakraborty [30] first reported the EOF of inelastic power-law fluids in slits. Zimmerman et al. [31] calculated numerically the electrokinetic flow of Carreau fluids in a T-shaped microchannel. The same model was used by Zhao et al. [32] in their investigation of steady EOF in a slit microchannel under the Debye–Hückel approximation. They obtained analytical solution of the velocity profiles which depend on the power law index n. Exact solutions for EOF of generalized fractional Oldroyd-B fluids in rectangular microchannels was obtained by Zhao and Yang [33] with the aid of finite Fourier and Laplace transforms. EOF of power-law fluids in a slit at high zeta potentials was studied by Vasu and De [34] and Zhao and Yang [35] respectively. Using lattice Boltzmann method, Tang et al. [36] simulated numerically EOF flow of Power-law fluids in microchannels. Very recently the extension to viscoelastic fluids was done by Park and Lee [37,38]. Afonso et al. [39] derived an analytical solution for the combined electroosmosis-Poiseuille flow of non-linear Phan–Thien–Tanner (PTT) and FENE-P models [40] in a two-dimensional channel. Dhinakaran et al. [41] extended the work of Ref. [39] by considering the full Gordon–Schowalter convective derivative in the PTT model.

Very recently, Liu et al. [42] first studied the time periodic EOF of generalized Maxwell fluids between two micro-parallel plates and an analytical solution of EOF velocity distribution is presented. The purpose of the present work is to derive an analytical solution for periodical electroosmotic flow of viscoelastic fluid in a complex two-dimensional rectangular microchannel. This work will be a good reference point for future work involving more sophistication or perhaps even as a benchmark for involved numerical work. This study will extend the one-dimensional slit case in Ref. [42] to two-dimensional rectangular microchannel case due to the practical cross-section shape of microfluidic devices.

2. Problem formulation

2.1. Cauchy momentum equation and constitutive relation

The unsteady EOF of the incompressible generalized Maxwell fluids through a two-dimensional rectangular microchannel of width $2W$ and height $2L$ with the z-axis being in the axial velocity direction is sketched in Fig. 1. The Cartesian axes are placed at the middle of the channel. The chemical interaction of electrolyte liquid and solid wall generates an EDL, a very thin charged liquid layer at the solid–liquid interface. The EOF flow is pumped by an axial (along z-direction) AC electric field of strength E_0, the liquid inside the EDL flows along the channel (in the direction orthogonal to the x–y plane) due to electroosmosis. For unidirectional flow, we consider the only axial velocity component $w(x, y, t)$ and the other velocity components in x–y plane disappear. Thus the incompressible condition is satisfied automatically.

Traditionally, in large-sized channels flow is often driven by pressure that is usually generated by mechanical pumps. In microchannels, however, it becomes increasingly difficult to utilize pressure-driven flow mode as the channel size shrinks, especially down to micro and submicrometer ranges. For an open-end horizontally placed channel, no pressure gradient is induced and hence the pressure gradient term in the Cauchy momentum equation disappears. In the z direction momentum equation, the pressure gradient along z direction is exactly zero. The two-dimensional Cauchy momentum equation can be expressed as

$$\rho\frac{\partial w(x,y,t)}{\partial t} = -\frac{\partial}{\partial x}(\tau_{xz}) - \frac{\partial}{\partial y}(\tau_{yz}) + \rho_e(x,y)E_z(t) \quad (1)$$

where $w(x, y, t)$ is the axial velocity, which is along positive z direction, ρ is the fluid density, t is time, τ_{xz} and τ_{yz} are the stress tensors, $\rho_e(x, y)$ is volume charge density and $E_z(t)$ is AC forced electric field. For generalized Maxwell fluids, the constitutive equation satisfies [43]

$$\tau_{xz} = -\int_{-\infty}^{t}\left\{\frac{\eta_0}{\lambda_1}\exp\left[-\frac{(t-t')}{\lambda_1}\right]\right\}\frac{\partial w(x,y,t')}{\partial x}dt' \quad (2a)$$

$$\tau_{yz} = -\int_{-\infty}^{t}\left\{\frac{\eta_0}{\lambda_1}\exp\left[-\frac{(t-t')}{\lambda_1}\right]\right\}\frac{\partial w(x,y,t')}{\partial y}dt' \quad (2b)$$

where λ_1 is the relaxation time, η_0 is the zero shear rate viscosity. Substituting Eq. (2) into Eq. (1) yields

$$\rho\frac{\partial w}{\partial t} = \int_{-\infty}^{t}\left\{\frac{\eta_0}{\lambda_1}\exp\left[-\frac{(t-t')}{\lambda_1}\right]\right\}\left[\frac{\partial^2 w(x,y,t')}{\partial x^2} + \frac{\partial^2 w(x,y,t')}{\partial y^2}\right]dt' + \rho_e(x,y)E_z(t) \quad (3)$$

Supposing the AC electric field and the velocity of the time periodical EOF of generalized Maxwell fluids can be written in complex forms as

$$w(x,y,t) = \Re\{w_0(x,y)e^{i\omega t}\}, \quad E_z(t) = \Re\{E_0 e^{i\omega t}\} \quad (4a,b)$$

where the $\Re\{\cdot\}$ denotes the real part of the function, ω is imposed AC electric field oscillating angular frequency. After substitution of Eq. (4) into Eq. (3), we can write

$$\rho\Re\{i\omega w_0 e^{i\omega t}\} = \Re\left\{\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2}\right)\int_{-\infty}^{t}\frac{\eta_0}{\lambda_1}\exp\left[-\frac{(t-t')}{\lambda_1}\right]e^{i\omega t'}dt'\right\} + \rho_e(x,y)\Re(E_0 e^{i\omega t}) \quad (5)$$

Making the change of variable $s = t - t'$, Eq. (5) becomes

$$\rho\Re\{i\omega w_0 e^{i\omega t}\} = \Re\left\{\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2}\right)e^{i\omega t}\int_{-\infty}^{t}\frac{\eta_0}{\lambda_1}e^{-\frac{s}{\lambda_1}}e^{-i\omega s}ds\right\} + \rho_e(x,y)\Re(E_0 e^{i\omega t}) \quad (6)$$

Next, we perform the integration over s:

$$\rho\Re\{i\omega w_0 e^{i\omega t}\} = \Re\left\{\frac{\eta_0}{1+i\lambda_1\omega}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2}\right)e^{i\omega t}\right\} + \rho_e(x,y)\Re(E_0 e^{i\omega t}) \quad (7)$$

Removing the real-operator sign from both sides, as well as the common multiplier $e^{i\omega t}$, to get

$$i\rho\omega w_0(x,y) = \frac{\eta_0}{1+i\lambda_1\omega}\left[\frac{\partial^2 w_0(x,y)}{\partial x^2} + \frac{\partial^2 w_0(x,y)}{\partial y^2}\right] + \rho_e(x,y)E_0 \quad (8)$$