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A new, unified, theoretical framework for the formulation of general, nonlinear, single-scale shell theories

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ABSTRACT

A new, general theoretical framework for the generation of single-scale shell theories is presented. The framework is developed for application to the analysis of the general laminated and monolithic shells. The proposed framework is intended to allow the accurate prediction of the effects of material nonlinearity and wave propagation effects through the thickness of a shell.

The starting point for the framework is a general, single-scale description of the displacement field expressed in terms of arbitrary expansion functions through the thickness of the shell. The functional forms and orders for the expansion functions in the displacement field representation are arbitrary. The development of the governing equations for the theory is carried out using the general nonlinear equations of continuum mechanics referenced to the initial configuration within the context of general coordinate systems. The equations of motion and the lateral surface boundary conditions for the theory are derived using the method of moments over the domain of the expansion functions. The (arbitrary) top and bottom surface boundary conditions (BCs) are satisfied exactly. The interfacial constraints (continuity of tractions and (dis) continuity of displacements) are also satisfied exactly. Delamination effects are incorporated into the theory through the use of arbitrary functions relating the displacement jumps to appropriate state variables. These functions can be changed without the need for reformulation of the governing equations. The theory is formulated in a sufficiently general fashion that any type of history-dependent material model can be used to describe the history-dependent behavior of the material composing a layer without the need to reformulate the theory.

The theoretical framework is unified in the sense that any type of desired single scale shell (smear/equivalent single layer (ESL), discrete layer, or zig-zag) theory can be obtained through suitable specialization of the framework. In the case of a smeared or ESL representation the domain of the displacement representation applies across the entire thickness of the shell. To generate a zig-zag theory within the context of the proposed framework is simply a matter of carrying out the interfacial analysis appropriate to the zig-zag assumptions and substituting the resulting displacement representation into the framework and then proceeding as with a smeared/ESL theory. In the case of a discrete layer analysis the displacement representations applies across each of the individual domains. The domains may correspond to several layers, a lamina, or a sublamina. Thus, the framework represents a comprehensive approach to modeling shells.

The predictions of the theory are compared with the results obtained from an exact elastic solution for the static response of a sphere and the exact elastic solution for the dynamic response of monolithic sphere. Both exact solutions are based on the assumptions of spherically symmetric boundary conditions. It is shown that the theory is capable of providing accurate predictions for the pointwise (displacement, strain, and stress) fields distributions in laminated and monolithic shells. Furthermore, it is shown that the behavior of the theory is self-convergent and thus increasing the order of the analysis always converges the predictions to the correct answer.

1. Introduction

Laminated shells are seeing an increased useage as load bearing components in advanced structural applications. Correspondingly, such advanced laminated shells are increasingly being subjected to ever more demanding dervice environments. Demanding service environments translate into demanding boundary conditions.

Often these BCs cover many orders of magnitude in rate and range at the low end from static conditions to, at the high end, high velocity impacts. In fact, it is common that these ranges in BCs

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exist simultaneously within the same structure. Typically, these boundary conditions are multiaxial and complex in nature.

There are two potential consequences of demanding BCs and service environments. The first consequence is that such conditions, typically, induce states of deformation and stress within the structure that are fully three-dimensional. The resulting fields can exhibit rapid changes in sign and magnitude. Severe deformations and complex states of stress usually result in history-dependent responses within the structure. These history-dependent responses, in turn, induce localization processes within the structure that lead to failure of the material and/or the structure in some sense. In any case, the effects of history-dependent phenomena on structural behaviors are becoming widely recognized (see for example [19,14,15]). The second consequence is that structures are being required to respond dynamically. Under these conditions wave propagation effects through the thickness of the shells can become important. For example, Whitney and Sun [37] show that higher order effects are necessary to correctly capture dynamic loading effects in laminated plates. Dynamic loading conditions can induce various forms of damage such as delamination and material microcracking/cracking.

Both of history-dependent phenomena and wave propagation effects are nonlinear phenomena with nonlinear evolutionary processes. Given such nonlinear phenomena and their nonlinear influences on the structural response, as well as the interactive nature of these phenomena, it is absolutely essential to have structural theories capable of accurately predicting the pointwise fields since relatively small inaccuracies in the predictions for the local fields can lead to significant and ever increasingly inaccurate predictions for the structural behavior [42,43,21,22,44,45]. Furthermore, since the above effects are mutually and nonlinearly interactive, any inability to correctly predict one of these effects accurately will directly impact the ability to predict the other phenomena accurately.

Shell theories have the potential to provide the requisite accurate predictions for the pointwise fields in the presence of both history-dependent phenomena and wave propagation effects needed to accurately model the behavior of advanced shell structures. Furthermore, these types of approaches have the potential to do this in a computationally efficient manner. However, meeting the requirement of accurately predicting pointwise behavior in shell structure places stringent constraints on what constitutes an appropriate shell theory for structural analyzes.

There are a wide variety of currently available shell theories. The simplest forms are the classical shell theories utilizing Kirchhoff-Love type assumptions [20,36,23,29]. Next in the scale of complexity are the so-called higher order shear deformation theories (for example see [37,27,33,30]). Attempts to develop theories with improved satisfaction of the interfacial conditions has lead to the development of the so-called to zig-zag (ZZ) theories (see for example [33,34,3,35]). More general displacement fields have been utilized in the development of discrete layer (DL) theories (see for example [1,31,32,2,4,9,10,13,38,28]). With respect to the above usage the term "single scale" refers the fact that the expansions are applied either to each layer or, alternatively, across the entire thickness of the laminate. A unified multiscale framework is given by Ref. [39]. In this case the term "multiscale" is used to denote the existence of two distinct scales (one associaed with the entire thickness plate and the other existing within the layers simultanesouly). It can be shown that these layer-wise and multiscale theories can be considered to be variationally-based, unified, single scale or multiscale frameworks [5,11,12,40]. As a consequence most if not all of the currently available, specialized, variatioally-derived, plate/shell theories can be obtained via appropriate specialization of one of these unified theories. Furthermore, methodologies, such as the use of genetic algorithms, for using these unified approaches to obtain the "best" possible theory for a given application have been discussed [6–8,13]. For additional comprehensive referencing of recent research on shells see the reviews given by Qatu [25], Qatu et al. [26], Reddy and Arciniega [30], Carrera [2].

Most currently available classical shell theories cannot be used for the analysis of advanced laminated structures that exhibit history-dependence and through the thickness wave propagation effects. This inability is due to the inherent limitations imposed by their initial assumptions. In particular, lower order theories (such as Kirchoff-type theories) cannot adequately reproduce the local pointwise fields and, thus, cannot correctly predict the nonlinear evolutionary processes associated with the previously discussed nonlinearities. Furthermore such theories are incapable of considering delamination processes due to the continuity of the displacement fields used in the underlying assumptions. Similar statements can be made with regard to higher order smeared/ESL theories. At this point in time it is unclear how to appropriately extend zig-zag theories to consider the effects of history-dependent phenomena. Additional considerations arise in the development of an appropriate shell theory capable of considering material nonlinearities in the form of delaminations. The unified approaches discussed above represent the best possible currently available analysis tools for these types of problems. However, under some circumstances where accurate assessments of the local fields is critical [16], it is possible that the weak formulations utilized in these theories can cause problems with accurate prediction of the behavior of laminated structures (without significantly increasing the computational demands).

The current work presents a new type of general, theoretical framework for modeling shell behavior. The framework can accurately predict pointwise fields and, therefore, accurately predict the effects of nonlinearities on the behavior of shells as well as being able to accurately predict wave propagation effects through the thickness of the shell. The development of the framework is not variationally based but rather utilizes a different formulational approach. The starting point for the framework is a general, single-scale description for the displacement field given in terms of arbitrary expansion functions through the thickness of the shell. The domain (s) of the single-scale representations are either the entire thickness of the shell (for a smeared theory or a zig-zag theory) or the thicknesses of the individual layers within the shells (a discrete layer theory) or mixed scale domains [13]. The basic displacement representation is completely general in terms of both orders and functional forms for the different effects. The theory satisfies the equations of motion and the lateral surface boundary conditions using the method of moments. The interfacial constraints and the top/bottom surface boundary conditions are satisfied exactly (i.e. in a strong sense). The theory is sufficiently general that any type of material constitutive equations can be incorporated into it without the need of reformulation. Delamination behavior is incorporated into the theory through the use of traction-displacement jump conditions at the internal interfaces between layers.

The predictions of the theory are compared with the results obtained from exact solutions for the static and dynamic behavior of shells. It is shown that the theory is capable of providing accurate predictions for the pointwise displacement, strain, and stress fields in laminated and monolithic shells to any desired level of accuracy for both static and dynamic loading states. This self-convergent attribute of the framework implies that the theory is capable of accurately predicting the evolution of nonlinear phenomena.

2. The General, Single-scale, Shell Framework (GSSSF)

Consider a laminated shell of arbitrary geometry composed of an arbitrary number of layers N, Fig. 1. A layer can consist of several