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Effect of mass diffusion on the damping ratio in a functionally graded micro-beam

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ABSTRACT

The present paper is aimed at studying the effects of mass diffusion on the quality factor of a functionally graded cantilever micro-beam in which the material properties of the micro-beam vary continuously along the beam thickness according to a power-law. The governing equation of a micro-beam deflection is obtained using Hamilton's principle and also the governing equations of thermo-diffusive elastic damping are established by using two dimensional non-Fourier heat conduction and non-Fickian mass diffusion models with one relaxation time based on continuum theory frame. The free vibration of the micro-beam resonators is analyzed by using Galerkin reduced order model formulation for the first mode of vibration. The mass diffusion effects on the quality factor are studied for the various micro-beam thicknesses and temperatures. Numerical computations are performed for specific materials and the results obtained are represented graphically. The effect of different power law exponent on the quality factor of the micro-beam is studied and Comparisons are made within the theory in the presence and absence of the mass diffusion effect.

1. Introduction

It has been observed that many materials have a spatially varying microstructure that leads to spatial variation in elastic properties and thus requires a nonhomogeneous model. Merging the most ideal properties of two different materials is realized in Functionally Graded Materials (FGM). A functionally graded material is characterized by a gradual change in material properties over volume. It is an anisotropic composite material where a material gradient has been deliberately introduced over two different materials. In contrast, traditional composites are homogeneous mixtures, and they therefore involve a compromise between the desirable properties of the component materials. Since significant proportions of an FGM contain the pure form of each component, the need for compromise is eliminated. The properties of both components can be fully utilized.

FGMs offer great promise in applications where the operating conditions are severe. For example, wear-resistant linings for handling large heavy abrasive ore particles, rocket heat shields, heat exchanger tubes, thermoelectric generators, heat-engine components, plasma facings for fusion reactors, and electrically insulating metal/ceramic joints. The mechanical and thermal response of materials with spatial gradients in composition and microstructure is of considerable interest in numerous technological areas such as tribology, optoelectronics, biomechanics, nanotechnology and high temperature technology. They are also ideal for minimizing thermomechanical mismatch in metal–ceramic bonding. Gradations in microstructure are also commonly found in biological cellular materials such as wood and bone, where biological adaptation has distributed the strongest microstructure in regions that experience the highest stress. Functionally graded materials is produced using advanced manufacturing techniques, including powder metallurgy, chemical vapor deposition, centrifugal casting, and so on.

In some cases, material gradation will reduce maximum stresses and change the spatial location where such maximums occur. This provides the possibility of tailoring material variation to achieve desired stresses in a structure and thus of functionally grading the material. The difficult part of this concept is how to develop manufacturing techniques that will produce the desired continuous modulus variation in realistic materials used in engineering applications.

The interest in graded materials focused primarily on the control of thermal stresses in elements exposed to high temperatures (to 1600 °C), for instance in gas turbine blades, aerospace structures, solid-oxide fuel cells, energy conversion systems using thermoelectric or thermionic materials (thermal barrier coatings, TBC). Subsequent applications include fusion and fast-breeder reactors as a first-wall composite material, piezoelectric and thermoelectric devices, high density magnetic recording media, in optical applications as graded refractive index materials in

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audio–video discs, in bioengineering as dental and orthopedic implants.

Elasticity formulation for inhomogeneous problems appeared in a half a century ago [1–4]. In the 1990s, interest in FGMs focused on thermal stresses. Recently, an elasticity solution is obtained for simply supported functionally gradient beams subjected to sinusoidal transverse loading by Sankar [5]. Exact solutions for bending and free vibration of functionally graded beams resting on a Winkler–Pasternak elastic foundation are presented based on the two-dimensional theory of elasticity by Ying et al. [6]. Li [7] presented a static result for a cantilever FGM micro-beam and also performed a dynamic analysis including wave propagation and free vibration. Kapuria et al. [8] studied experimentally, as well as theoretically the static and free vibration response of layered FGM beams of Al/SiC and Ni/Al2O3 prepared with powder metallurgy and combustion powder thermal spray processes, respectively. Simsek and Kocatürk [9–11] investigated non-linear dynamic behavior of an FG beam with pinned–pinned ends subjected to a moving harmonic load by using Timoshenko beam theory with the effect of the geometric non-linearity. Mahi et al. [12] studied the free vibration analysis of symmetric FGM beam subjected to initial thermal stresses with an analytical method.

Today the development of mechanical and electronic systems in a compact case such as Micro-Electro-Mechanical-Systems (MEMS) has become more important, because this essential part of technology increases the speed and compacts size of industrial equipment. In fact, MEMS is a combination of mechanical components, electronic sensors and electronic components.

The term MEMS first started being used in the 1980s. It was used in the United States for first time and has been applied to a broad set of technologies with the goal of miniaturizing systems through the integration of functions into small packages. This mechanical structure is very small in size, and has various applications. MEMS have grown tremendously in the last decade and have attracted worldwide attention. This is due to the wide application of MEMS in different branches of medicine, medical engineering (analysis and synthesis of DNA and genetic code, drug delivery, diagnostic and imaging), transportation systems (converters, accelerometers, Gyroscopes) and production (intelligent micro-robots) and etc.

Micro-mechanical resonators are a category of MEMS devices, these resonators used widely in applications requiring high speed and accuracy. Recently developing and manufacturing mobile devices with limited energy source to reduce energy consumption has been inevitable and this is available by analyzing all factors, interfering in energy consumption in these devices.

To obtain high performance resonators, it is necessary to design and build a resonator that works with very low power dissipation or in other words, it works with a high quality factor. Quality factor of the resonator is a measure of the amount of energy loss.

Air and squeeze-film damping and clamping losses are the major extrinsic mechanisms of energy dissipation. Air and squeeze-film damping, can be avoided by using vacuum packaging. Damping due to dislocation motion and grain boundary sliding, can be avoided by using homogeneous Single Crystalline Silicon (SCS) as the structural material, and by ensuring that stress levels are sufficiently low. Recent studies have shown that when slender structures with micro-scale dimensions are vibrated in bending, thermoelastic damping (TED) occurs. Energy loss resulting from the thermoelastic damping is the main intrinsic mechanism of energy dissipation factors in micro-beam resonators [13,14].

As the name thermoelastic suggests, it describes the coupling between the elastic field in the structure caused by deformation and the temperature field. In precise measurements, TED acts as a source of mechanical Thermal noise and contributes to reduce the quality factor and this causes an increase in energy consumption. Therefore it is important to find a way to reduce intrinsic losses, such as TED, as much as possible [15–19].

Firstly, Zener [20] identified the existence of TED as a significant dissipation mechanism in flexural resonators. Lifshitz and Roukes [21] studied an exact solution of thermoelastic dissipation for resonator beams and their results showed that the simplified classical results of Zener [20] is very close to the exact solution under reasonably fair conditions. Sun et al. [22] studied and analyzed the TED of micro-beam resonators by using both the finite sine Fourier transformation method combined with Laplace transformation and the normal mode analysis.

Mohammadi-Alasti et al. [23] discussed about the mechanical behavior of a functionally graded micro-beam subjected to a thermal moment and non-linear electrostatic pressure and they investigated the behavior of reverse FGM micro-beam versus voltage and temperature changes. Vahdat and Rezazadeh [24] studied the effects of axial and residual stresses on TED in capacitive micro-beam resonators. Vahdat et al. [25] investigated the TED in a micro-beam resonator tunable with a pair of piezoelectric layers bonded on its upper and lower surfaces. Their investigation results demonstrate that thickness of the piezoelectric layers and application of DC voltage to them can affect the TED ratio and the TED critical thickness (thickness that relates to the maximum damping ratio) value of the resonator.

Mass Diffusion (MD) defined as the random movement, of an ensemble of particles, from regions of higher concentration to lower concentration regions. Recently there is an increasingly attention on the study of MD phenomenon because of its applications in electronic industries.

In integrated circuit manufacture, diffusion is used to introduce dopants in controlled quantity into the semiconductor substance. Especially, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors and the source/drain regions in Metal Oxide Semiconductor (MOS) transistors, and dope poly-silicon gates in MOS transistors [26].

In most of the previous investigations, the mass concentration is calculated using a simple law what is known as Fick's equation. Until recently, thermodiffusion in solids was considered as a quantity that is independent of the body deformation. Study of the phenomenon of MD shows that the process of thermodiffusion could have a very important effect upon the deformation of the body.

Nowacki [27–30] developed the theory of thermoelastic diffusion by using a coupled thermoelastic model. Sherief et al. [26] developed the generalized theory of thermoelastic diffusion with one relaxation time, which allows the finite speeds of propagation of waves. Sherief and Saleh [31] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time.

The above explanations declare that mass diffusion damping (MDD) is the other important intrinsic loss mechanism and also the advantage of FGM micro-beam is behind its response due to thermal and diffusivity loading. The question arises whether the previous works are still valid for design high quality factor resonators and whether the diffusivity can be separated from FGM micro-beam resonator analysis or the MDD effect has to be taken into account. In many cases however many researches performed their researches on FGM materials without considering diffusivity condition of the problem but in most cases it is obvious that using functionally graded materials will not only improve mechanical properties of the structure but also makes it worse (if there is not diffusivity loading).

As far as we know researchers have not attended on the effect of MDD in the FGM micro-beam resonators and consequently the papers published in this field are far from reality and are not applicable in precise measurement scales. As should be evident, problem formulation is more challenging therefor the aim of the