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Instability of eccentrically stiffened functionally graded truncated conical shells under mechanical loads

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ABSTRACT

This paper is concerned with the mechanical buckling load of an eccentrically stiffened truncated conical shells made of functionally graded materials and subjected to axial compressive load and external uniform pressure by analytical method. Shells are reinforced by stringers and rings. The change of spacing between stringers in the meridional direction is taken into account. Material properties of shell are graded in the thickness direction according to a volume fraction power-law distribution. The equilibrium and linearized stability equations for stiffened shells are derived based on the classical shell theory and smeared stiffeners technique. The resulting equations which they are the couple set of three variable coefficient partial differential equations in terms of displacement components are investigated by Galerkin method and the closed-form expression for determining the buckling load is obtained. The effects of stiffeners, material and dimensional parameters are analyzed in detail.

1. Introduction

A conical shell is one of the common structural elements used in modern airplane, missile, booster and other aerospace vehicles. As a result, problems relating to stability and vibration of these structures have received considerable attention of researchers. Seide [1,2] investigated the buckling of conical shells under the axial loading. Singer [3] presented the buckling of conical shells subjected to the axisymmetrical external pressure. Lu and Chang [4] and Chang and Lu [5] examined the themoelastic buckling of conical shells based on linear [4] and nonlinear [5] analyses. They used Galerkin method for integrating the equilibrium equation. Tani and Yamaki [6] obtained the results of truncated conical shells under axial compression.

Using the Donnell-type shell theory, the linear buckling analysis of laminated conical shells, with orthotropic stretching-bending coupling, under axial compressive load and external pressure, are studied by Tong and Wang [7]. The same authors [8] investigated linear buckling analysis of laminated composite conical shells. Eight first-order differential equations were obtained and solved by the numerical integration technique and the multisegment method in their work. Wu and Chiu [9] presented a three-dimensional solution for the thermal buckling of the laminated composite conical shells.

Xu et al. [10] presented a solution for nonlinear free vibration of symmetrically laminated cross-ply conical shell with its two ends both clamped and both simply supported. Galerkin procedure and the method of harmonic balance used to analyze dynamic responses of shell. Lam et al. [11] reported an improved generalized differential quadrature method for the investigation of the effects of boundary conditions on the free vibration characteristics of truncated conical panels. The effects of the vertex angles on the frequency parameters were examined in their study. Based on the classical thin shell theory, Liew et al. [12] considered the free vibration analysis of thin conical shells under different boundary conditions using the element-free kp-Ritz method. The kernel particle (kp) functions are employed in hybridized form hamonic functions to approximate the two-dimensional displacement field. Civalek [13] proposed a discrete singular convolution method for analyzing the free vibration of rotating conical shells. A regularized Shannon's delta kernel is selected as the singular convolution to illustrate his algorithm.

In recent years, because functionally graded material (FGM) structures are widely used in modern engineering, the stability and vibration behaviors of FGM plates and shells have attracted increasing research effort. Among those available, Sofiyev [14–16] investigated the linear stability and vibration of unstiffened FGM truncated conical shells with different boundary conditions. The same author [17] presented the nonlinear buckling behavior and nonlinear vibration [18] of FGM truncated conical shells, and considered [19] the buckling of FGM truncated conical shells subjected to axial compressive load and resting on Winkler–Pasternak foundations. For linear analysis, the general characteristics in his works is that the modified Donnell-type equations are used and Galerkin method is applied to obtain closed-form relations of

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bifurcation type buckling load or to find expressions of fundamental frequencies, whereas for nonlinear analysis, the large deflection theory with von Karman–Donnell-type of kinetic nonlinearity is used.

Based on the first-order shell theory by Love–Kirchhoff and the Sanders nonlinear kinetic equations, the thermal and mechanical instability of FGM truncated conical shells also is investigated by Naj et al. [20]. Bich et al. [21] presented results on the buckling of un-stiffened FGM conical panels under mechanical loads. The linearized stability equations in terms of displacement components are derived by using the classical shell theory. Galerkin method is applied to obtained explicit expression of buckling load. Malekzadeh and Heydarpour [22] studied the influences of centrifugal and Coriolis forces in combination with the other geometrical and material parameters on the free vibration behavior of rotating FGM truncated conical shells subjected to different boundary conditions based on the first-order shear deformation theory. As can be seen that the above introduced works only relate to unstiffened structures. However, in practice, plates and shells including conical shells, usually reinforced by stiffeners system to provide the benefit of added load carrying capability with a relatively small additional weight. Thus, the study on static and dynamic behavior of theses structures are significant practical problem. Weingarten [23] conducted a free vibration analysis for a ring-stiffened simply supported conical shell by considering an equivalent orthotropic shell and using Galerkin method. He also carried out experimental investigations. Crenwelge and Muster [24] applied an energy approach to find the resonant frequencies of simply supported ring-stiffened, and ring and stringer-stiffened conical shells. Mustaffa and Ali [25] studied the free vibration characteristics of stiffened cylindrical and conical shells by applying structural symmetry techniques. Some significant results on vibration of FGM conical shells, cylindrical shells and annular plate structures with a four-parameter power-law distribution based on the first-order shear deformation theory are analyzed by Tornabene [26] and Tornabene et al. [27].

Srinivasan and Krisnan [28] obtained the results on the dynamic response analysis of stiffened conical shell panels in which the effect of eccentricity is taken into account. The integral equation for the space domain and mode superposition for the time domain are used in their work. Based on the Donnell–Mushtari thin shell theory and the stiffeners smeared technique, Mecitoglu [29] studied the vibration characteristics of a stiffened truncated conical shell by the collocation method. The minimum weight design of axially loaded simply supported stiffened conical shells with natural frequency constraints is considered by Rao and Reddy [30]. The influence of placing the stiffeners inside as well as outside the conical shell on the optimum design is studied. The expressions for the critical axial (buckling) load and natural frequency of vibration of conical shell also are derived.

For stiffened FGM structures, recently there are some investigations which have been focused on the analysis of the static buckling and postbuckling, vibration and dynamic buckling of plates and shells. Najafizadeh et al. [31] with the linearized stability equations in terms of displacements studied buckling of FGM cylindrical shell reinforced by rings and stringers under axial compression. The stiffeners and skin, in their work, are assumed to be made of functionally graded materials and its properties vary continuously through the thickness direction. Bich et al. [32] presented an analytical approach to investigated the nonlinear post-buckling of eccentrically stiffened FGM plates and shallow shells based on the classical shell theory in which the stiffeners are assumed to be homogeneous. Dung and Hoa [33] obtained the results on the static nonlinear buckling and post-buckling analysis of eccentrically stiffened FGM circular cylindrical shells under external pressure. The material properties of shell and stiffeners are assumed to be continuously graded in the thickness direction. Galerkin method was used to obtain closed-form expressions to determine critical buckling loads. Bich et al. [34] obtained the results on the nonlinear dynamic analysis of eccentrically stiffened FGM cylindrical panels. The governing equations of motion were derived by using the smeared stiffeners technique and the classical shell theory with von Karman geometrical nonlinearity. The same authors [35] investigated the nonlinear vibration dynamic buckling of eccentrically stiffened imperfect FGM doubly curved thin shallow shells based on the classical shell theory. The nonlinear critical dynamic buckling load is found according to the Budiansky–Roth criterion.

The review of the literature signifies that few analytical studies have been carried out to investigate on the stability of FGM conical shells and there is no work on the analytical solution for combined loaded stiffened FGM conical shells. This may be attributed to the inherent complexity of governing equations of conical shell. Those are variable coefficient partial differential equations. In addition, for stiffened conical shell, the spacing between stringers in the meridional direction also varies. These difficulties have to be got over as investigating the stability of truncated conical shells. In this paper, a mechanical buckling of eccentrically stiffened functionally graded (ES-FGM) thin truncated conical shells subjected to axial compressive load and uniform external pressure load is investigated. The present novelty is that the shells under combined load are reinforced by rings and stringers attached to their inside or outside. The change of spacing between stringers in the meridional direction is taken into account. The material properties of shell are graded continuously in the thickness direction. The theoretical formulations based on the smeared stiffeners technique and the classical shell theory, are derived. The resulting equations which they are the couple set of three variable coefficient partial differential equations in terms of displacement components are solved by Galerkin method. The closed-form expressions to determine critical buckling loads are obtained. The influences of various parameters such as stiffener, dimensional parameters and volume fraction index of materials on the stability of shell are clarified in detail.

2. FGM truncated conical shell and theoretical formulation

2.1. Functionally graded truncated conical shell

Consider a thin truncated conical shell of thickness h and semi-vertex angle α. The geometry of shell is shown in Fig. 1, where L is the length and R is its small base radius. The truncated cone is referred to a curvilinear coordinate system (x, θ, z) whose the origin is located in the middle surface of the shell, x is in the generatrix direction measured from the vertex of conical shell, θ is in the circumferential direction and the axes z being perpendicular to the plane (x, θ), lies in the outwards normal direction of the cone. Also, x_0 indicates the distance from the vertex to small base, and u, v, and w denote the displacement components of a point in the middle surface in the direction x, θ and z, respectively.

Assume that the truncated conical shell is made from a mixture of a ceramic and a metal (denoted by c and m, respectively) and the material compositions only vary smoothly along its thickness direction with the power law distribution as

$$V_m + V_c = 1, \quad V_c = V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^k \tag{1}$$

where $-h/2 \leqslant z \leqslant h/2$ and $k \geqslant 0$ is the volume fraction and takes only non-negative values.

According to the mentioned law, the Young's modulus can be expressed by

$$E = E(z) = E_m V_m + E_c V_c = E_m + E_{cm}\left(\frac{2z + h}{2h}\right)^k \tag{2}$$