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Thermo-electro-mechanical vibration of piezoelectric nanoplates based on the nonlocal theory

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ABSTRACT

This paper mainly studies the thermo-electro-mechanical free vibration of piezoelectric nanoplates based the nonlocal theory and Kirchhoff theory. It is assumed that the piezoelectric nanoplate is a simply supported rectangular plate, and subjected to a biaxial force, an external electric voltage and a uniform temperature change. The governing equations and boundary conditions are derived by using the Hamilton's principle, which are then solved analytically to obtain the natural frequencies of the piezoelectric nanoplate. A detailed parametric study is conducted to discuss the influences of the nonlocal parameter, axial force, external electric voltage and temperature change on the thermo-electro-mechanical vibration characteristics of piezoelectric nanoplates.

1. Introduction

Piezoelectric materials can be found of various practical applications in smart structures and systems owning to their intrinsic electromechanical coupling effect [1,2]. Extensive technical literatures regarding the macroscopic piezoelectric materials can be found in the monographs [3,4]. As for the piezoelectric nanomaterials, Pan et al. [5] first reported the ZnO piezoelectric nanowires in *Science* magazine. Since then, a variety of piezoelectric nanomaterials (e.g. ZnO, ZnS, PZT, GaN, $BaTiO_3$, etc.) and their nanostructures (e.g. nanowires, nanobelts, nanorings, nanohelices, etc.) received considerable attentions from research communities and institutes [6–9]. The piezoelectric nanostructures are of significant thermal, electrical, mechanical and other physical/chemical properties compared with their macro-scale counterparts [6,9], and of potential applications in many nanodevices [10–13], including nanoresonators, nanogenerators, light-emitting diodes, chemical sensors, etc.

It should be pointed out that the dimension of piezoelectric nanostructures may vary from several hundred nanometers to just a few nanometers. On this scale, the size effect becomes very obvious and essential, which has already been proved by many experiments and atomistic simulations [14,15]. However, the classical continuum theory, which is a scale-independent theory, fails to meet the computing demands of nanostructures. So, extensive high-order theories such as the strain gradient theory, couple stress theory, micro-polar theory, and nonlocal elasticity theory, are developed to modify the classical continuum theory and to characterize the size effect of nanostructures by introducing an intrinsic length scale. Among all these high-order theories, the nonlocal elasticity theory raised by Eringen [16–18] is generally accepted and applied to analyze the scale effect of nanostructures. By considering the interactions and forces between atoms, the nonlocal theory introduces the internal length scale into the constitutive equations as a material parameter. Based on the nonlocal theory, the analysis of the size-dependent properties of nanostructures becomes an active research recently. Meanwhile, the nonlocal nanobeam, nanoplate and nanoshell models were developed to solve the bending [19–21], buckling [22,23], linear and nonlinear vibrations [24–27], postbuckling [28] and wave propagation [29,30] of nanostructures.

The above studies mainly concerned the size effect on the elastic nanostructures like carbon nanotubes, graphene sheets, etc. Recently, the nonlocal theory was extended to the piezoelectric nanostructures by Ke and his co-authors and Arani and his co-authors. Ke and Wang [31] and Ke et al. [32] analyzed the linear and nonlinear vibrations of piezoelectric nanobeams based on the nonlocal theory and Timoshenko beam theory. On the foundation of nonlocal piezoelectric shell theory, Arani et al. [33,34] analyzed the axial buckling and free vibration of double-walled Boron Nitride nanotubes (BNNTs) embedded in an elastic medium under combined electro-thermo-mechanical loads. And also, except for the nonlocal effect, the surface effect on the piezoelectric nanostructures has also concerned by many investigators. Huang and Yu [35] examined the effect of the surface piezoelectricity on the electromechanical behavior of the piezoelectric ring. According to

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Huang and Yu's surface piezoelectricity model, Yan and Jiang [36–39] considered the bending, buckling and vibration behaviors of piezoelectric nanobeams and nanoplates.

In this paper, we investigate the thermo-electro-mechanical free vibration of piezoelectric nanoplates based on nonlocal theory and Kirchhoff plate theory. It is assumed that the piezoelectric rectangular nanoplate is of all edges simply supported, and subjected to a biaxial force, an external electric voltage and a uniform temperature change. The governing equations and boundary conditions are derived by using the Hamilton's principle, which are then solved analytically to obtain the natural frequency of the piezoelectric nanoplate. The influences of the nonlocal parameter, biaxial forces, external electric voltage and temperature change on the thermo-electro-mechanical vibration characteristics of piezoelectric nanoplates are examined.

2. Nonlocal theory for the piezoelectric materials

According to the nonlocal elasticity theory developed by Eringen [16–18], the stress at a point $\mathbf{x}$ in a body depends not only on the strain at that point but also on those at all other points $\mathbf{x}'$ of the body, which is in accordance with the atomic theory of the lattice dynamics and the experiment of phonon dispersion [17]. By introducing a nonlocal attenuation function to account for the effect of long-range interatomic forces, the nonlocal elasticity theory explains satisfactorily some phenomena such as the high frequency vibration and wave dispersion. Mathematically, ignoring the effect of body force, the basic equations for a homogeneous and nonlocal piezoelectric solid can be expressed as

$$\sigma_{ij} = \int_V \alpha(|\mathbf{x}' - \mathbf{x}|, \tau)[c_{ijkl}\varepsilon_{kl}(\mathbf{x}') - e_{kij}E_k(\mathbf{x}') - \lambda_{ij}\Delta T]\mathrm{d}\mathbf{x}', \quad (1)$$

$$D_i = \int_V \alpha(|\mathbf{x}' - \mathbf{x}|, \tau)[e_{ikl}\varepsilon_{kl}(\mathbf{x}') - \kappa_{kij}E_k(\mathbf{x}') + p_i\Delta T]\mathrm{d}\mathbf{x}', \quad (2)$$

$$\sigma_{ij,j} = \rho\ddot{\mathbf{u}}_i, \quad D_{i,j} = 0, \quad (3)$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\Phi_i, \quad (4)$$

where V is the volume of the piezoelectric solid; σ_{ij}, ε_{ij}, D_i, E_i and u_i are respectively the components of the stress, strain, electric displacement, electric field and displacement; c_{ijkl}, e_{kij}, κ_{kij}, λ_{ij}, p_i and ρ are respectively the elastic constants, piezoelectric constants, dielectric constants, thermal moduli, pyroelectric constants and mass density; ΔT and Φ are the temperature change and electric potential, respectively. $\alpha(|\mathbf{x}' - \mathbf{x}|,\tau)$ represents the nonlocal attenuation function, incorporating into the constitutive equations the influences at the reference point produced by the local strain at the source $\mathbf{x}'$, where $|\mathbf{x}' - \mathbf{x}|$ is the Euclidean distance. $\tau = e_0a/l$ is the scale coefficient that incorporates the small scale factor, where e_0 is a material constant determined experimentally or approximated by matching the dispersion curves of the plane waves with those of the atomic lattice dynamics; and a and l are the internal (e.g. lattice parameter, granular size) and external characteristic lengths (e.g. crack length, wavelength) of the nanostructures, respectively.

However, the above expressions are difficult in solving results mathematically due to the spatial integrals in the constitutive equations. According to Eringen [17], the integral constitutive relations is converted to an equivalent differential form as

$$\sigma_{ij} - (e_0a)^2\nabla^2\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \lambda_{ij}\Delta T, \quad (5)$$

$$D_i - (e_0a)^2\nabla^2 D_i = e_{ikl}\varepsilon_{kl} - \kappa_{kij}E_k + p_i\Delta T, \quad (6)$$

where ∇^2 is the Laplace operator; e_0a is the scale coefficient revealing the size effect on the response of structures in nanosize.

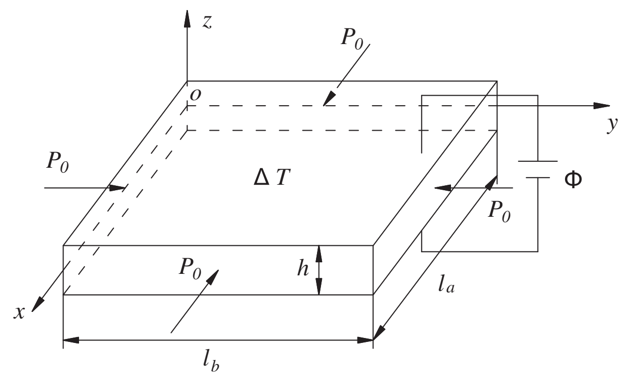

Fig. 1. A piezoelectric nanoplate under thermo-electro-mechanical loadings.

3. Free vibration analysis of the piezoelectric nanoplate

In this section, the free vibration of the piezoelectric nanoplate under thermo-electro-mechanical loadings is analyzed on the basis of the nonlocal theory for the piezoelectric materials mentioned above. As is shown in Fig. 1, consider a rectangular piezoelectric nanoplate with length l_a, width l_b and thickness $h$under the plane stress state, defined in the rectangular coordinate system ($0 \leqslant x \leqslant l_a$, $0 \leqslant y \leqslant l_b$, $-h/2 \leqslant z \leqslant h/2$). The nanoplate is subjected to a biaxial force P_0 (compressive force or tensile force), an applied voltage $\Phi(x, y, z, t)$ and a uniform temperature change ΔT. The poling direction of the piezoelectric medium is parallel to the positive z-axis. According to the Kirchhoff plate theory, the displacement of an arbitrary point along the x-, y- and z-axis, denoted as $u_1(x, y, z, t)$, $u_2(x, y, z, t)$ and $u_3(x, y, z, t)$ can be expressed as

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) - z\frac{\partial w(x,y,t)}{\partial x}, \\ u_2(x,y,z,t) &= v(x,y,t) - z\frac{\partial w(x,y,t)}{\partial y}, \\ u_3(x,y,z,t) &= w(x,y,t), \end{aligned} \quad (7)$$

where $u(x, y, t)$ and $v(x, y, t)$ are the in-plane displacements of the mid-plane in the nanoplate (on the x- and y-direction, respectively); $w(x, y, t)$ is the out-plane displacement of the mid-plane in the nanoplate (on the z-direction) and t is the time.

In addition to the displacement field, the distribution of the electric potential should conform to the Maxwell equation. In Fig. 1, the top and bottom surfaces of the piezoelectric plate have electrodes to facilitate the application of voltage to actuate the structure. When an external voltage is applied, the electric potential distribution on the surface of the electrode remains constant. When electrodes at the two surfaces of the piezoelectric plate are shortly connected, the electric potential is zero throughout the surfaces [40–42]. Quek and Wang [40] and Wang [43] performed the dispersion and buckling characteristics of the piezoelectric plate and beam by assuming the electric potential as a combination of a cosine and linear variation in order to satisfy the Maxwell equation. This assumption is verified numerically by finite element method for the case of a uniform moment applied to the piezoelectric plate [42]. Therefore, following Quek and Wang [40] and Wang [43], the electric potential is approximately assumed as a combination of cosine and linear variation,

$$\Phi(x,y,z,t) = -\cos(\beta z)\phi(x,y,t) + \frac{2zV_0}{h}e^{i\Omega t}, \quad (8)$$

where $\beta = \pi/h$; $\phi(x, y, t)$ is the spatial and time variation of the electric potential in the mid-plane; V_0 is the external electric voltage; Ω is the natural frequency of the piezoelectric nanoplate.