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Finite element analysis of laminated composite and sandwich shells using higher order zigzag theory

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ABSTRACT

Static analysis of laminated composite and sandwich shell is presented by developing a C_0 finite element (FE) formulation based on higher order zigzag theory (HOZT) using Sander's approximations. The proposed model satisfies the inter-laminar shear stress continuity at the interfaces; and zero transverse shear stress conditions at shell top and bottom. This is the first finite element implementation of the HOZT to solve the problem of shells incorporating cross curvature effects in shells. The present HOZT predicts transverse shear stresses more accurately than FE results previously published in literature. Numerical results show that the present 2D model is very efficient in predicting the static response of laminated composite and sandwich shallow shell very close to 3D elasticity solutions.

1. Introduction

Laminated composite and sandwich shell structures are widely used in civil, mechanical, aerospace and other engineering applications. Laminated composite and sandwich materials are becoming popular because of their high strength to weight and strength to stiffness ratio. The most important feature for the analysis of laminated composite and sandwich structure is that the material is weak in shear compared to extensional rigidity. Due to this reason transverse shear deformation of the composite and sandwich shell has to be modeled very efficiently.

Classical theories originally developed for thin elastic shells are based on the Love–Kirchhoff assumptions [1]. These theories neglect the effect of transverse shear deformations. Therefore, application of such theories to laminated composite and sandwich shells, where shear deformation is very significant, may lead to errors in calculating deflections, stresses and frequencies. In subsequent development of shell theories transverse shear deformation was included in a manner where the shear strain is constant throughout the thickness of the shell. These theories are known as first order shear deformation theory (FSDT). In this theory shear correction factors are required for the analysis and these factors should be calculated based on the orientations of different layers in different directions. Love's First Approximation Theory [1] is inconsistent with respect to rigid body motions. Sanders' theory [2] removed this inconsistency. In Sanders' theory all strains vanish for small rigid-body motions of the shell hence Sanders' theory is more close to actual behavior of shells than Love's theory.

The effects of transverse shear and normal stresses in shells were considered by Hildebrand [3], Reissner [4]. The first thin shell theory for laminated orthotropic material was developed by Ambartsumyan [5]. Ambartsumyan [5] extended Reissner–Mindlin theory to layered, anisotropic plates and shells. The bending–stretching coupling due to unsymmetric lamination in composites is also included in this theory. The effect of transverse shear deformation and transverse isotropy, as well as thermal expansion through the thickness of cylindrical shells were considered by Gulati and Essenberg [6], Ambartsumyan [7], Zukas and Vinson [8], Dong and Tso [9] and Whitney and Sun [10,11]. Ambartsumyan [7] was among the first to present a technique with transverse shear stress continuity conditions across the laminate interfaces. Ambartsumyan [7] shear deformation theory for the analysis of composites considered the parabolic distribution for the transverse shear stresses in a composite layer.

The higher-order shell theories presented in [10,11] are based on a displacement field in which the in-plane displacements of the shell are expanded as linear functions of the thickness coordinate and the transverse displacement is expanded as a quadratic function of the thickness coordinate. These higher-order shell theories are computationally more demanding, since with each additional power of the thickness co-ordinates, an additional dependent unknown is introduced into the theory.

Reddy and Liu [12] presented a simple higher-order shear deformation theory (HSDT) for the analysis of laminated shells. It contains the same dependent unknowns as in the first-order shear deformation theory (FSDT) in which the displacements of the middle surface are expanded as linear functions of the thickness

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coordinate and the transverse deflection is assumed to be constant through the thickness. The theory is based on a displacement field in which the in-plane displacements of the middle surface are expanded as cubic functions of the thickness coordinate and the transverse displacement is assumed to be constant through the thickness. The additional dependent unknowns introduced with the quadratic and cubic powers of the thickness coordinate are evaluated in terms of the derivatives of the transverse displacement and the rotations of the normals at the middle surface. This displacement field leads to the parabolic distribution of the transverse shear stresses (and zero transverse normal strain) and no shear correction factors are used. Kant and Menon [13] presented higher-order theories for composite and sandwich cylindrical shells but it contained some parameters which have no physical significance, hence it is difficult to incorporate the boundary conditions. Huang [14] presented modified Reddy's theory and further improved the accuracy. Yang [15] developed a higher-order shell element with three constant radii of curvature, two principal radii, orthogonal to each other and one twist radius. The displacement functions *u*, *v* and *w* are composed of products of one-dimensional Hermite interpolation formulae. However, these theories demand C_1 continuity of transverse displacements during their finite element implementations.

Shu and Sun [16] developed an improved higher-order theory for laminated composite plates. This theory satisfies the stress continuity across each layer interface and also includes the influence of different materials and ply-up patterns on the displacement field. Liew and Lim [17] proposed a higher-order theory by considering the Lame' parameter $(1 + z/R_x)$ and $(1 + z/R_y)$ for the transverse strains, which were neglected by Reddy and Liu [12]. This theory accounts for cubic distribution (non-even terms) of the transverse shear strains through the shell thickness in contrast with the parabolic shear distribution (even-terms) of Reddy and Liu [12]. Bhimaraddi [18], Mallikarjuna and Kant [19], Cho et al. [20] are among the others to develop higher-order shear deformable shell theory. It is observed that except for the theory of Yang [15], remaining higher-order theories do not account for twist curvature $(1/R_{xy})$, which is essential while analyzing shell forms like hypar and conoid shells.

Qatu et al. [21] reviewed most of the research done in recent years (2000–2010) on the static and buckling behavior of composite shells. It is observed application of higher-order theory for studying the behavior of laminated composite and sandwich shells with the combination of all three radii of curvature is very limited in literature. Pradyumna and Bandyopadhyay [22] studied the behavior of laminated composite shells based on a higher-order shear deformation theory (HSDT) developed by Kant and Khare [23]. However, this theory [23] contains some nodal unknowns which are not having any physical significance and therefore, incorporation of appropriate boundary conditions becomes a problem.

The typical feature of sandwich panel is that the variation of in-plane displacements across the thickness shows kink at the interface between the core and face layer, which gives discontinuous transverse shear strain at these interfaces. This phenomenon is also found in laminated composite plates for its layered construction but the discontinuity is not so prominent as that of sandwich shell. Actually, it depends on the difference in the values of transverse shear rigidity and thickness of adjacent layers, which is quite significant in sandwich shell compared to that of composite laminates. However, the effect of this discontinuity cannot be ignored in a multi layered thick composite laminate. It is observed that increase in the order of variation for the in-plane displacements across the thickness (i.e., HSDT), cannot make any significant improvement in representing the behavior of multi-layered laminates. Actually, it gives a continuous variation of shear strain across the thickness, which gives discontinuity in the shear stress distribution at the layer interfaces due to different values of shear rigidity at the adjacent layers. But the actual phenomenon is just opposite, i.e. the shear strain is discontinuous and shear stress is continuous at the layer interfaces. In order to consider this aspect, the layer-wise theories are proposed. In these theories the unknowns are taken at all the layer interfaces, which give a zigzag pattern of through thickness variation for the in-plane displacement to represent the desired shear strain discontinuity at the layer interfaces. In terms of solution accuracy, the performance of these theories is very good but they require a huge computational effort as the number of unknowns increase with the increase in the number of layers.

Noor et al. [24,25] focuses that the CLT and FSDT models as well as any higher-order smeared-laminate model based on an overall approximation of the in-plane displacements across the thickness fail to yield accurate results when sandwich composites are (i) thick, (ii) the ratio of the transverse shear modulus to in-plane modulus is low, (iii) anisotropy is severe, and (iv) the ratios of longitudinal to transverse Young's moduli are high. However, it should be emphasized that the assumptions of these models are based on making them simple yet limit their validity, especially with modern sandwich panels, and in some cases even lead to erroneous results. It is concluded that such description would require the use of layer-wise or zig-zag models.

Carrera [26] compared more than fifty available theories and finite elements to those developed in the framework of the unified notation. Carrera [26] addressed closed form solutions and finite element results, zig-zag effects and interlaminar continuity in laminated plates and shells. Carrera and Ciuffreda [27] uses a unified formulation to compare about 40 theories for multilayered, composites and sandwich plates which are loaded by transverse pressure with various inplane distributions. Carrera et al. [28] considered the Mixed Interpolation of Tensorial Components (MITC) technique to develop shear locking free refined multilayered plate elements. Cinefra et al. [29] used nine nodes shell finite element employing refined models based on the CUF (Carrera Unified formulation) for the static analysis of plates and shells made of functionally graded material. Cinefra and Carrera [30] analyzed the linear static analysis of composite cylindrical structures by means of refined models grouped in CUF employing a nine nodes shell finite element with variable through-the-thickness kinematic.

Shu [31] presented a closed form analytical solution based on a shell theory in combination with Love's first-order geometric approximation and Donnell's simplification for shallow composite shells. The theory improves the in-plane displacement (*u*, *v*) distribution through thickness by ensuring the continuity of interlaminar transverse shear stresses and zero transverse shear strains on the surface. The theory contains the same dependent unknown and the same order of governing equations as in the first-order shear deformation theory. Without the need for shear correction factors, the theory predicts more accurate responses than first-order theory and some higher-order theories, and the solutions are very close to the results of the elasticity solutions.

It is also observed that there is no literature available on finite element implementation of composite and sandwich shells based on higher order zigzag theory incorporating all three radii of curvatures.

Considering all the above aspects, a new C_0 finite element shell 2D model based on Sanders' theory [2] and Ambartsumyan [7] theory is developed. Further, the present formulation also includes cross curvature effects for the static analysis of composite and sandwich shell panels. This theory may be called as higher order zigzag theory (HOZT).

It gives parabolic through thickness variation of transverse shear strains with discontinuity at the layer interfaces as desired. It also gives zero shear strains at the top and bottom surfaces of