Composite Structures 106 (2013) 350–361

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

# Vibration analysis of functionally graded rectangular plates resting on elastic foundation using higher-order shear and normal deformable plate theory

S.A. Sheikholeslami, A.R. Saidi *

*Department of Mechanical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran*

ARTICLE INFO

*Article history:*
Available online 24 June 2013

*Keywords:*
Free vibration
Functionally graded
Elastic foundation
Higher-order shear and normal deformable plate theory
Analytical solution

ABSTRACT

The free vibration of simply supported functionally graded rectangular plates resting on two-parameter elastic foundation is studied using the higher-order shear and normal deformable plate theory of Batra and Vidoli by an analytical approach. All three displacement components are expanded in the thickness direction using the Legendre polynomials. The effects of transverse shear and normal deformations are considered and the equations of motion are derived using the principle of virtual work. A power law distribution is used to explain the variation of mechanical and physical properties through the thickness of the functionally graded plate. Governing equations are then derived and the natural frequencies and the corresponding mode shapes are obtained up to the fifth-order expansion. The numerical results are given in detail and compared with the existing works. It is shown that when fifth-order expansion is used, the results of this theory for natural frequencies of thick functionally graded rectangular plates are very close to those obtained from three dimensional elasticity theory.



## 1. Introduction

Due to very good property of functionally graded (FG) materials in resisting against high temperature gradients, the industrial application of this new class of materials is growing. Thermal resistance properties of ceramics and the high toughness of metals allow us to combine them and get a new multi-objective material. In this type of non-homogeneous (or heterogeneous) materials, mechanical properties, such as density and the Lamé coefficients, vary smoothly from one material point to another. This superior property of FG plates prevents the stress concentration through the thickness, which occurs in laminated composite plates.

Different theories are used for the analysis of plates in accordance to their thickness-to-length ratio. The simplest theory, which is called the classical plate theory (CPT), neglects the effects of shear and normal deformations in the thickness direction. Although this theory gives good results for thin plates with the thickness-to-length ratios of about 1/20, it fails to predict the appropriate solutions when this ratio increases. For analysis of moderately thick plates, the shear deformation theories are used. The first-order shear deformation theory (FSDT) assumes the variation of shear strain to be linear through the thickness of the plate. In general, the shearing stress on the top and bottom surfaces of the plate may vanish and, therefore, a shear correction factor is needed. Note that this theory considers the transverse shearing strain but not the normal one. In the higher-order shear and normal deformable plate theory (HOSNDPT), the effects of both shear and normal deformations are considered. The theory is called higher-order if the terms of $z^K$ ($K \geqslant 2$) are kept in the Taylor series expansions of displacement field components in the $z$-direction (through-the-thickness coordinate).

Among several works done recently on the static and dynamic analysis of FG rectangular plates without considering elastic foundation, some are of great interest. Reddy [1] presented the formulation and finite element models for static and dynamic analysis of FG plates using the third-order shear deformation theory. Ferreira et al. [2] analyzed the free vibration of FG plates based on the first and third-order shear deformation theories using the Mori–Tanaka homogenization method and the global collocation method with multiquadratic radial basis functions. Batra and Vidoli [3] used Legendre polynomials in $z$ as basis functions and derived a higher-order shear and normal deformable plate theory for piezoelectric plates from a three-dimensional variational principle. Free vibration and stress distribution of isotropic plates with HOSNDPT were analyzed by Batra and Aimmanee [4] using the finite element method. Batra [5] derived the HOSNDPT for an incompressible functionally graded plate. However, a homogeneous and isotropic plate was presumed in order to get the governing equations. Free and forced vibration of thick rectangular plates using HOSNDPT

* Corresponding author. Tel.: +98 341 2111763; fax: +98 341 2120964.
*E-mail address:* saidi@uk.ac.ir (A.R. Saidi).

0263-8223/$ - see front matter 
http://dx.doi.org/10.1016/j.compstruct.2013.06.016

and meshless local Petrov–Galerkin (MLPG) method was studied by Qian et al. [6]. In spite of these works, there are some other 2-D higher-order theories like the one derived by Matsunaga [7]. The major difference between Matsunaga's theory and the HOSNDPT is that in the former theory, the displacement field components are expanded in the $z$-direction using Taylor's expansion and, therefore, the monomials 1, $z$, $z^2$, ..., $z^K$ as basis functions, while the latter theory uses the Legendre polynomials as the basis functions for expanding the displacement field components in the thickness direction.

The works on the vibration of FG plates are not limited to the various two-dimensional theories, a three-dimensional exact solution for vibration of FG rectangular plates was given by Vel and Batra [8].

Because of the widespread applications of foundations in engineering, the interaction between engineering components and elastic media has been considered in many researches recently. The mechanical behavior of one-parameter elastic foundation was first discussed by Winkler [9], whereas Pasternak [10] considered a two-parameter model in analyzing elastic foundations. Kerr [11] presented various models for elastic and viscoelastic foundations. Lam et al. [12] used Green's functions and presented canonical exact solutions for bending, buckling and vibration of Levy-type plates on elastic foundation. Buckling and vibration results for thin rectangular plates on Pasternak foundations were given by Takahashi and Sonoda [13]. A refined plate theory which accounted for a quadratic variation of the transverse shear strains across the thickness of functionally graded plates resting on elastic foundation was developed by Thai and Choi [14]. Matsunaga[15] derived a two-dimensional higher-order theory for simply supported rectangular homogeneous plates on elastic foundations. Based on third-order shear deformation theory, Baferani et al. [16] presented an accurate solution for free vibration of functionally graded thick rectangular plates resting on elastic foundation. Dehghan and Baradaran [17] solved the eigenvalue equations based on a mixed finite element (FE) and differential quadrature (DQ) method to obtain the natural frequency and buckling load parameters.

Exact solutions based on the three-dimensional theory of elasticity for functionally graded thick plates resting on elastic foundation were presented by Huang et al. [18]. Amini et al. [19] investigated three-dimensional free vibration analysis of functionally graded material plates resting on an elastic foundation. Malekzadeh [20] adopted a semi-analytical approach using the differential quadrature method and series solution to analyze free vibration of FG rectangular plates on elastic foundations.

To the best of authors' knowledge, there is no analytical work on the free vibration of functionally graded rectangular plates resting on elastic foundation based on the higher-order shear and normal deformable plate theory.

In this paper, using the higher-order shear and normal deformable plate theory of Batra and Vidoli [3], the free vibration of simply supported FG rectangular plates resting on two-parameter elastic foundation is investigated analytically. The numerical results are presented up to fifth-order expansion and they are compared with those presented in the literature for some special cases. This theory gives good results for thick FG rectangular plates resting on elastic foundation.

## 2. Formulation of the problem

Using Cartesian coordinates, the infinitesimal deformations of an isotropic functionally graded plate resting on elastic foundation are described. Assume $h$ to be the plate thickness and let the $x_1x_2$-plane coincide with the mid-plane. The geometry and coordinates of the plate are shown in Fig. 1.

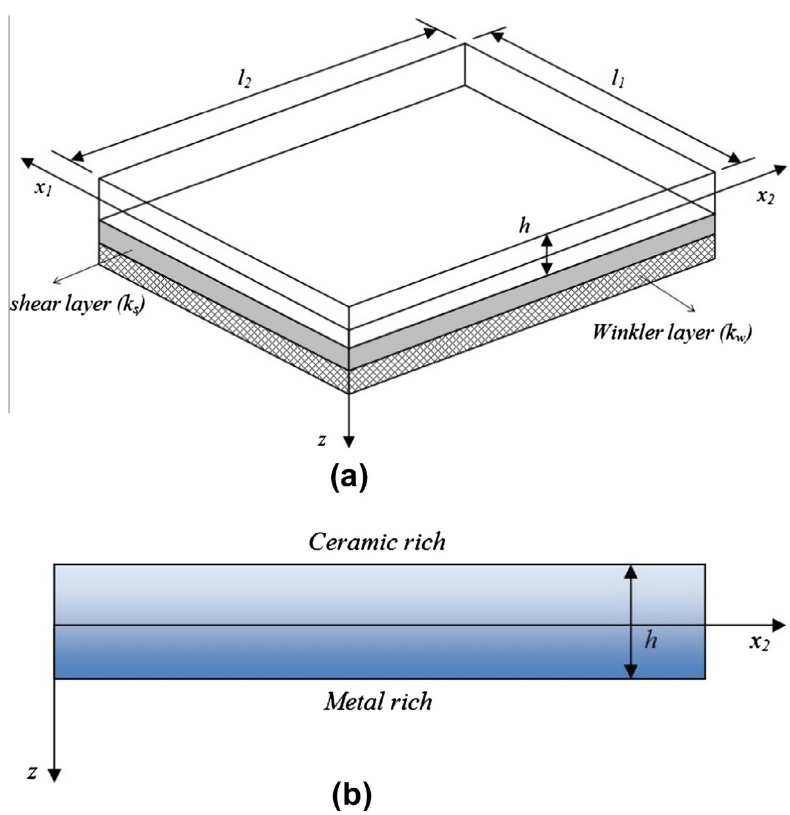


**Fig. 1.** The geometry and coordinates of FG rectangular plate.