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Axiomatic/asymptotic evaluation of multilayered plate theories by using single and multi-points error criteria ☆

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ABSTRACT

This paper deals with refined theories for multilayered composites plates. Layer-Wise (LW) and Equivalent Single Layer (ESL) theories are evaluated by means of axiomatic–asymptotic approach. Theories with forth order displacement fields in the thickness layer/plate direction z are implemented by referring to the Unified Formulation by Carrera. The effectiveness of each term of the made expansion is evaluated by comparing the related theories with a reference solution. As a result a reduced model is obtained which preserve the accuracy of the full-model (model that include the whole terms of the z-expansion) but it removes the not-significant terms in the same expansion (those terms that do no improve the results according to a given error criteria). Various single-point and multi-point error criteria have been analyzed and compared in order to establish such an effectiveness: error localized in an assigned point along z, error localized at each interface, error located at the z-value corresponding to the maximum value of the considered variables, etc. Applications are given in case of closed form solutions of orthotropic cross-ply, rectangular, simply supported plates loaded by bisinusoidal distribution of transverse pressure. Symmetrically and unsymmetrical laminated cases are considered along with sandwich plates. It is found the reduced model is strongly influenced by the used localized error and that in same case the reduced model which is obtained by of single point criteria can be very much improved by the use of multi-point criteria.

1. Introduction

The analysis of refined theories for anisotropic, multilayered composite structures is a topic of interest since decades. Classical theories based on the extension of developments originally made for homogenous one-layered beams (as Euler–Bernoulli [1–3]) or plates (Kirchhoof [4] and Reissner–Mindlin [5,6]) show severe limitation to analyze laminated composites structures. In particular these classical approaches are not able to describe so-called zig-zag field for the through-the-thickness distribution of displacement variables as well as interlaminar continuity of transverse both shear and normal stresses. These points were summarized in [7] as C_z^0-requirements, that is displacement and transverse stress field must be C^0 function along the z-thickness coordinate. Many theories are known that permit to overcome the limitation of classical theories, review papers on that topics are those by Ambartsumian [8,9], Librescu and Reddy [10], Kapania and Raciti [11], Noor and Burton [12,13], Librescu [14] and in particular the historical note by Carrera [15]. In general better description is obtained by referring to so-called Layer-Wise (LW) theories with respect to Equivalent Single Layer (ESL) ones. Each layer is considered as an independent plate in the case of LW theories while the number of the unknown variables remains independent by the number of the layers in the ESL approaches.

Among the many available techniques the present work refer to those based on Carrera Unified Formulation (CUF) which has been successfully introduced over the last decade. According to CUF it is possible to express the governing equations in terms of so-called fundamental nuclei whose form does not depend on either the expansion order, nor on the choices made for the base functions. More details on CUF can be found in [16–18]. ESL and LW approaches were successfully developed in the mainframe of CUF theory. In [19] ESL and LW models up to fourth N-order expansion in the thickness layer/plate direction were considered. The conclusion of that study was that LW approach could provide a realistic

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description of transverse stresses of laminate thick and thin plate while ESL approach accuracy depends on the laminate lay-out. Thermomechanical analysis of simply supported multilayered plates employing ESL and LW models was addressed in [20]. Applications to sandwich structures was given in [21]. Comparison of ESL and LW approaches can be found in [22] where the authors investigated the linearized buckling of laminated plates. Recent dynamic analysis have been provided in [23].

Due to its features CUF represents an interesting framework to compare and assess advanced theories. In particular it could be used to establish the accuracy of a given ESLM or LW theories with a given order of the expansion for the displacement variables, in case of displacement formulated theories. CUF as definition should be classified as axiomatic theories, that is the order of the expansion for the displacement variables is assumed ‘a priori’. It is well known that in contrast to axiomatic approach the asymptotic approach could be used. The latter expand the governing equations in terms of a perturbation parameter δ of the structures (e.g. the length-to-thickness ratio) by leading to class-of-problems related to set governing equations which contain the whole contribution with the same order of magnitude with respect to δ. Reviews and analysis on this approach with applications to plates and shells can be found in [24–30].

By using axiomatic approach it has been shown that the introduction of high order terms in a model offers a benefit in terms of improved structural response analysis, as a drawback higher computational cost is requested. The possibility to obtain accurate high order theories with less computational cost could be offered by evaluating the importance/effectiveness of each term of the expansion in the solution process. With that information a decision could be taken and the corresponding term could be retained (if relevant) or discarded (if not significant). By doing that the so-called *asymptotic/axiomatic* technique is obtained: the effectiveness of each displacement variable of a model is compared to a reference solution and the terms which do not influence the response are discarded. This technique was proposed in the finite element framework in [31–33]. The genetic like algorithms were used in [34] to evaluate the importance of each displacement variables for FE plate models. The results of these works were presented in diagram. That diagram was stated as ‘Best Plate Theories curves’; it gives the minimum number of displacement variables versus the accuracy on a given stress or displacement parameter.

Closed form solution related to axiomatic/asymptotic technique have been proposed in [35]. The authors analyzed isotropic, orthotropic and composite plates considering different parameter (i.e. *a*/*h* ratios, orthotropic ratios and ply sequence) and the best models

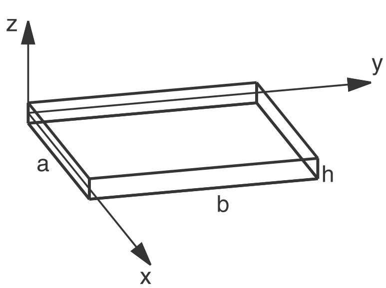

Fig. 1. Plate reference system.

Table 1
Symbols to indicate the status of a displacement variable.

Active term	Inactive term	Interface terms in LD4 (always active)
▲	△	■

Table 2
Example of representation of the reduced kinematics model with u_{z2} deactivated.

■	▲	▲	▲	■
■	▲	▲	▲	■
■	△	▲	▲	■

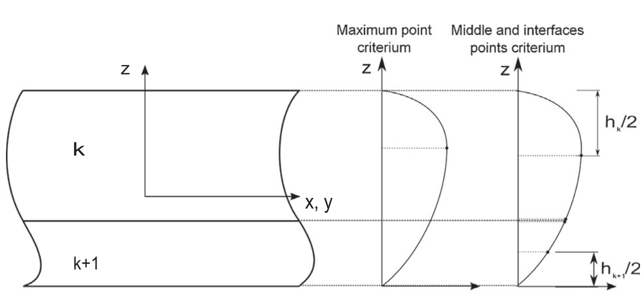

Fig. 2. Maximum point, interfaces and middle-layer points criterium.

Table 3
Comparison of ED4 and LD4 results with the exact solutions by Pagano [38]. Material data: $E_L/E_T = 25$, $(G_{LT}, G_{Lz})/E_T = 0.5$, $G_TT/E_T = 0.2$, $\nu_{LT} = \nu_{Lz}\nu_{TT} = 0.25$. Displacement $\bar{u}_z = u_z 100 E_T h^3/(p_0 a^4)$ evaluated at $z = 0$.

	3D	ED4	LD4
	$a/h = 4$		
Cylindrical bending cases			
$N_l = 3$	2.887	2.685	2.887
$N_l = 4$	4.181	3.830	4.180
	$a/h = 6$		
$N_l = 3$	1.635	1.514	1.635
$N_l = 4$	2.556	2.362	2.556
	$N_l = 3$		
	3D	ED4	LD4
Bisinusoidal loading cases, $b = 3a$			
$a/h = 4$	2.887	2.625	2.821
$a/h = 10$	0.919	0.867	0.919

Table 4
Composite square plate under bisinusoidal loadings. $\bar{u}_z = \frac{100}{p_z}\frac{u_z}{a^4}\frac{E_T}{}\frac{h^3}{}$, $(\bar{\sigma}_{xz}, \bar{\sigma}_{yz}) = (\sigma_{xz}, \sigma_{yz})/(p_0\ a/h)$, $\bar{\sigma}_{xx} = \sigma_{xx}/(p_0\ (a/h)^2)$.

	$\bar{u}_z(z = h/2)$	$\bar{\sigma}_{xx}(z = h/2)$	$\bar{\sigma}_{xz}(z = 0)$	$\bar{\sigma}_{yz}(z = 0)$	$\bar{\sigma}_{zz}(z = h/2)$
	$a/h = 100$, 0°/90°/0°				
CLT	0.2836	0.5637	0.3280	0.0239	0.8448
FSDT	0.2852	0.5634	0.3279	0.0240	0.8465
ED4	0.2854	0.5639	0.4542	0.0449	0.0100
LD4	0.2854	0.5639	0.4054	0.0727	0.0100
	$a/h = 100$, 0°/90°/0°/90°				
CLT	0.3374	0.0239	0.1736	0.1447	0.9798
FSDT	0.3383	0.0239	0.1736	0.1447	0.980
ED4	0.3387	0.0240	0.2683	0.2235	0.0349
LD4	0.3387	0.0239	0.2848	0.2848	0.0100
	$a/h = 2$, 0°/90°/0°				
CLT	0.2828	0.5625	0.3277	0.0238	0.0169
FSDT	3.1598	0.3536	0.2255	0.1040	0.0456
ED4	5.3692	1.2074	0.2687	0.1632	0.5091
LD4	5.5094	1.4089	0.2541	0.2000	0.5025
	$a/h = 2$, 0°/90°/0°/90°				
CLT	0.3374	0.0239	0.1736	0.1447	0.0196
FSDT	2.6402	0.0239	0.1736	0.1447	0.0196
ED4	5.2633	0.1753	0.2325	0.2029	0.5105
LD4	5.3511	0.1786	0.2065	0.2650	0.5012