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Free vibration analysis of laminated composite shallow shells with general elastic boundaries

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ABSTRACT

A unified and exact solution method is developed for the free vibration analysis of composite laminated shallow shells with general elastic boundary conditions, a class of problem which is of practical interest and fundamental importance but rarely attempted in the literature. Under the current framework, each of the shell displacements, regardless of boundary conditions, is expanded as a standard Fourier cosine series supplemented with closed-form auxiliary functions introduced to eliminate all the relevant discontinuities with the displacement and its derivatives at the edges and accelerate the convergence of series representations. Mathematically, such series expansions are capable of representing any functions (including the exact displacement solutions). Rayleigh–Ritz procedure is employed to obtain the exact solution based on the energy functions of the shell. The current method can be universally applicable to a variety of boundary conditions including all the classical cases, elastic restraints and their combinations. The excellent accuracy and reliability of the current solutions are validated by numerical examples, and the effects of boundary restraining stiffnesses and lamination schemes on frequency parameters are illustrated. New results for various shell curvatures including spherical, cylindrical and hyperbolical paraboloidal shells with elastically restrained edges are presented, which may serve as benchmark solutions.

1. Introduction

Shallow shells are open shells that have small curvatures (i.e. large radii of curvatures compared with other shell parameters such as length and width). With the progress of composite materials, shallow shells constructed by composite laminas are extensively used in many fields of modern engineering practices requiring high strength-weight and stiffness-weight ratios such as aircraft structures, space vehicles and deep-sea engineering equipments. It is noticed that the laminated composite shallow shells in these applications often work in complex environmental conditions and suffer to non-classical boundary conditions. Therefore, a complete understanding of the bucking, bending, vibration, impact and other characteristics of these shells with non-classical boundary conditions is of particular importance.

Laminated composite shallow shells can be formed as rectangular, triangular, trapezoidal, circular or any other planforms and various types of curvatures such as singly-curved (e.g., cylindrical), double-curved (e.g., spherical, hyperbolic paraboloidal) or other complex shapes such as turbomachinery blades. Moreover, laminated composite plates can be treated as the special cases of the shallow shells. In recent decades, a huge amount of research efforts have been devoted to the vibration analysis of laminated shallow shells, aiming to provide insight into dynamic behaviors and optimal design of complex composite shallow shells. So far, some of the static and dynamic behaviors of these shells with classical boundary conditions had being presented precisely. Some research papers and articles oriented to such contributions may be found in following enumeration. A meshfree method based on the wavelet collocation is used by Ferreira et al. [1] to compute the static deformations and frequencies of doubly-curved composite shells. Fazzolari and Carrera [2] developed a hierarchical trigonometric Ritz formulation for free vibration and dynamic response analysis of doubly-curved anisotropic laminated shallow and deep shells. The static and free vibration analysis of doubly-curved shells was performed by Ferreira et al. [3,4] with radial basis functions collocation. Exact solutions of the equations and fundamental frequencies for simply supported, doubly-curved, cross-ply laminated shells were presented by Reddy and Asce [5]. A generalized modal approach is applied by Khdeir and Reddy [6] to predict free and force vibration of cross-ply laminated composite shallow arches. Natural vibration of completely free laminated composite triangular and trapezoidal shallow shells was studied by Qatu [7]. Using the five-degrees-of- freedom shallow shell theory, the stress anal-

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ysis of cross-ply laminated plates and shallow shell panels having a rectangular plan-form was accurately predicted by Soldatos and Shu [8]. Some other contributors in this subject are Librescu et al. [9,10], Dogan and Arslan [11], Leissa and Chang [12], Chakravorty et al. [13], Oktem and Chaudhuri [14], Kurpa et al. [15], Ghavanloo and Fazelzadeh [16], Chakavorty et al. [17], Viola et al. [18,19], Singh and Kumar [20], Qatu [21–24] and others [25–32]. More detailed and systematic summarizations can be seen in the excellent monographs by Qatu [33–36] and Reddy [37].

The above researches show the extensive interest in vibration behaviors of laminated composite shallow shells. However, it is evident that the information available about the vibration of laminated composite shallow shells is far from complete. Most of the previous studies on this subject are confined to the classical boundary conditions (i.e., simply-supported, clamped, free and shear-diaphragm boundaries) and their combinations. However, in many engineering applications, the boundary conditions of a shallow shell may not always be classical in nature. A variety of possible boundary restraining cases such as elastic edge conditions, non-uniform boundaries, point-supported supports and other complicated boundary conditions may be encountered in practices. These problems have received very limited attention amongst the researchers. The only work is available in the open literature is that of Chakavorty et al. [17], who used eight-node curved quadrilateral isoparametric finite elements to studying the free vibration behavior of point-supported laminated shallow shell and that of Narita and Leissa [38], that presented the vibration analysis of corner point-supported isotropic spherical and hyperbolic paraboloidal shells with equal principal radii of curvature. Moreover, the existing solution methods are often only customized for a specific set of different boundary conditions, and thus typically require constant modifications of the trial functions and corresponding solution procedures to adapt to different boundary cases. Therefore, the use of the existing solution procedures will result in very tedious calculations and be easily inundated with various boundary conditions in practical applications because even only considering the aforementioned classical (homogeneous) cases, one will have a total of hundreds of different combinations. It is necessary and of great significance to develop an efficient method which is capable of universally dealing with composite shallow shells with arbitrary boundary conditions. However, very limited literature can be found on the topic. Recently, using algebraic functions, Qatu and Asadi [39] presented the first comprehensive free vibration study of isotropic shallow shells subjected to arbitrary boundary conditions by Ritz method.

The objective of this paper is to fill the analytical gap of elastically restrained laminated shallow shells and develop a unified, efficient and accurate solution for free vibration analysis of arbitrary laminated composite shallow shells with general elastic boundary conditions in rectangular planform. The present work can be considered as an extension of the modified Fourier series method previously developed [40–43] for vibration analysis of isotropic beams and plates with general elastic supports. An arbitrary laminated version of the Donnel–Mushtari's theory which is accurate for shallow shells [33,39,44] is employed to formulate the theoretical model. Each of three displacements of the laminated composite shallow shell, regardless of boundary conditions, is expanded as a standard Fourier cosine series supplemented with several auxiliary functions. The introducing of the auxiliary functions is to remove any potential discontinuities of the original displacements and their derivatives throughout the entire shell structure including the boundaries and then to effectively enhance the convergence of the results. Since the displacement field is constructed adequately smooth throughout the entire solution domain, an exact series solution is obtained by using Rayleigh–Ritz procedure based on the energy functions of the shell. The accuracy and reliability of current solutions are validated by several numerical examples, and the effects of restraining stiffnesses and lamination schemes on frequency parameters are investigated. For the sake of presenting the proposed solution clearly, the laminated composite shallow shells considered in the following discussions are focused on rectangular planforms. The proposed solution can be readily applied to shallow shells with other planforms.

2. Theoretical formulations

2.1. Description of the model

Consider an elastically restrained, laminated composite thin shallow shell in rectangular planform as shown in Fig. 1, the length, width and thickness of the shell are represented by a, b and h, respectively. The middle surface of the shell where an orthogonal coordinate (x, y and z) is fixed is taken as the reference surface of the shell. The shallow shell is characterized by its middle surface, which can be defined by [33]:

$$z = -\left(\frac{x^2}{2R_x} + \frac{xy}{R_{xy}} + \frac{y^2}{2R_y}\right) \tag{1}$$

where R_x and R_y represent the radii of curvature in the x, y directions as depicted in Fig. 1. R_{xy} is the corresponding radius of twist. In present work, we focus on the cases when R_x, R_y and R_{xy} are constants. In addition, the x and y coordinates are conveniently oriented to be parallel to boundaries so that $R_{xy} = \infty$. The displacements of the shell in the x, y and z directions are denoted by u, v and w, respectively. As presented in Fig. 1, along each end of the shell, three groups of translational springs (k_u, k_v and k_w) and one group of rotational springs (K_x) are distributed uniformly along the boundary to separately simulate the given or typical boundary conditions expressed in the form of axial force resultant, tangential force resultant, transverse force resultant, and the flexural moment resultant. Specifically, k^u_{x0}, k^v_{x0}, k^w_{x0} and K^w_{x0} denote the set of springs distributed along the edge $x = 0$. Similarly, by replacing the subscript $x0$ with xa, $y0$ and yb, the other three sets of continuously distributed boundary springs at corresponding ends can be designated respectively. Thus, the given boundary conditions can be readily achieved by assigning the translational and rotational springs at proper stiffnesses. The classical boundary conditions can be seen as special cases of elastic support conditions. For instance, a clamped boundary condition can be generated by simply setting the stiffnesses of all the springs equal to infinite (which is represented by a very large number, 10^{12} N/m). Inversely, a free boundary is gained by setting the stiffnesses of all the springs equal to zero.

In Fig. 2, the shallow constructed as various types of curvatures are plotted. It can be plate (i.e. $R_x = R_y = R_{xy} = \infty$), spherical shell (e.g., $R_x = R_y = R$, $R_{xy} = \infty$), circular cylindrical shell (e.g., $R_x = R$, $R_y = R_{xy} = \infty$) and hyperbolic paraboloidal shell (e.g., $R_x = -R_y = R$, $R_{xy} = \infty$).

2.2. Kinematic relations and stress resultants

According to the Kirchhoff hypothesis and Qatu's formulation [33], the middle surface strains and curvature changes of the considered shallow shell can be written in terms of displacements as:

$$\begin{aligned} &\varepsilon_{0x} = \frac{\partial u}{\partial x} + \frac{w}{R_x}, \quad \varepsilon_{0y} = \frac{\partial v}{\partial y} + \frac{w}{R_y}, \quad \gamma_{0xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} \\ &\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_y = -\frac{\partial^2 w}{\partial y^2}, \quad \kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{2}$$

where ε_{0x}, ε_{0y} and γ_{0xy} indicate the strains in middle surface; κ_x, κ_y and κ_{xy} are the curvature changes. Thus, the liner strains in the kth layer space of the shallow shell are defined as: