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Variational asymptotic micromechanics modeling of heterogeneous magnetostrictive composite materials ☆

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ABSTRACT

A new micromechanics model is developed to predict the effective properties as well as the local fields of heterogeneous magnetostrictive composite materials using the variational asymptotic method for unit cell homogenization (VAMUCH), a recently developed micromechanics modeling technique. Starting from the total magnetic enthalpy of the heterogenous continuum, we formulate the micromechanics model as a constrained minimization problem taking advantage of the fact that the size of the microstructure is small compared to the macroscopic size of the material. To handle realistic microstructures in engineering applications, we implement this new model using the finite element method. A few examples are used to demonstrate the application and accuracy of the proposed theory and the companion computer program-VAMUCH.

1. Introduction

Magnetostrictive materials such as $CoFe_2O_4$ are widely used in sensors and actuchanical energy, and vice versa. Howevercks for instance their weight, disadvantage of shape control, and acoustic impedance, therefore magnetostrictive composite materials are usually a better technical solution in the case of many applications such as ultrasonic generators, ultrasonic receivers, and echo detectors. Recently, magnetostrictive composites are developed by combining magnetostrictive materials with passive materials to form a variety of types of magnetostrictive composite systems. To facilitate the design and analysis of such materials, convenient and accurate analysis tools are apparently indispensable.

Although it is logically sound to use the well-established finite element method (FEM) to analyze such structures by meshing all the details of constituent materials, the size of the finite element model will easily overpower most of the computers we can access in the foreseeable future because the macroscopic structural dimensions are usually several orders of magnitude larger than the characteristic size of constituent materials.

In the past several decades, numerous approaches have been proposed to predict the effective properties of magnetostrictive composites from known constituent information. Simple analytical approaches based on Voigt or Reuss hypothesis have been applied to predict the behavior of a limited class of composite geometries [1]. Variational bounds have been obtained for describing the complete overall behavior which are useful tools for theoretical consideration [2]. However, the range between bounds could be too large to be of practical use. Researchers have proposed various techniques to either reduce the difference between the upper and lower bounds, or find an approximate value between the upper and lower bounds. Typical approaches are the self-consistent approach [3], mathematical homogenization theories (MHTs) [4], finite element approaches using conventional stress analysis of a representative volume element (RVE) [5]. Li and Dunn [6] employed the Mori–Tanaka method [7] for predicting the average fields and effective moduli of fully coupled magneto-electric–elastic properties of circular cylinder fibrous and laminated two-phase composites. Other studies have focused on the classical extensions of Eshelbys solutions [8] (mean field-type methods) for finite inclusion volume fraction, i.e., the differential approaches [9] and models based on the generalized Mori–Tanaka and self-consistent approaches [10].

The objective of this paper is to develop a micromechanics model based on the framework of variational asymptotic method for unit cell homogenization (VAMUCH) for predicting the effective properties and local fields of magnetostrictive composites. This framework is build upon the variational asymptotic method (VAM) [11] along with two essential assumptions within the concept of micromechanics for composite with an identifiable Unit Cell (UC). VAM simplifies the procedure of solving physical problems that can be formulated in terms of a variational statement involving one or more small parameters, which has been used

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Nomenclature

v_i, ψ^m	the global displacement and global magnetic displacement, respectively
u_i, ϕ^m	the displacement field and magnetic potential field defined in the integer space, respectively
$\tilde{u}_i$, $\tilde{\phi}^m$	the displacement field and magnetic potential field defined in the 3D space, respectively
Ω	a volume occupied by the UC
h	characteristic size of the UC
l	characteristic wavelength of the deformation of the structure
σ_{ij}, ε_{ij}	stress tensor and strain tensor, respectively
H_i, B_i	magnetic field vector and magnetic displacement vector, respectively
H	magnetic enthalpy
D	matrix containing all the necessary material constants
C	elastic constants matrix
q	piezomagnetic coefficients matrix
μ	magnetic permeability matrix
λ_i, λ^m, α_{ij}, β_i	Lagrange multiplies introduced to enforce the constraints
x	global coordinate to describe the macroscopic structure
y	local coordinate to describe the UC
n	integer-valued coordinate to locate a UC in the heterogeneous material
w_i, w^m	the fluctuation functions
S	the shape functions
V	a column matrix of the nodal values of fluctuation functions
d_i	dimension of a UC
S_i	surfaces with $n_i = 1$
R	3D space
$\overline{D}$	effective (or homogenized) material matrix

extensively to construct efficient high-fidelity structure models for composite beams [12], composite and smart plates [13,14], and composite and smart shells [15,16], achieving an excellent compromise between accuracy and efficiency. The two essential assumptions are:

Assumption 1. The exact field variables have volume averages over the UC. For example, if u_i and ϕ^m are the exact displacements and magnetic potential within the UC occupying a volume Ω, there exist v_i and ψ^m such that (here and throughout the paper, Latin indices assume 1, 2, and 3 and repeated indices are summed over their range except where explicitly indicated.)

$$v_i = \frac{1}{\Omega}\int_\Omega u_i \mathrm{d}\Omega \equiv \langle u_i \rangle \tag{1}$$

$$\psi^m = \frac{1}{\Omega}\int_\Omega \phi^m \mathrm{d}\Omega \equiv \langle \phi^m \rangle \tag{2}$$

Assumption 2. The effective material properties obtained from the micromechanical analysis of the UC are independent of the geometry, the boundary conditions, and loading conditions of the macroscopic structure, which means that effective material properties are assumed to be the intrinsic properties of the material when viewed macroscopically.

Note that these assumptions are not restrictive. The mathematical meaning of the first assumption is that the exact solutions of the field variables can be integrated over the domain of UC, which is true almost all the time and the very basic requirement for us to perform the homogenization. The second assumption implies that we will neglect the size effects of the material properties in the macroscopic analysis, which is an assumption often made in the conventional continuum mechanics necessary for the definition of material properties. Of course, the micromechanical analysis of the UCs is only needed and appropriate if $h/l \ll 1$ (h as the characteristic size of the UC and l as the characteristic wavelength of the deformation of the structure). Other assumptions such as particular geometry shape and arrangement of the constituents, specific boundary conditions applied to the UC, and prescribed relations between local fields and global fields are not necessary for the present study.

In VAMUCH, both the multiscale asymptotic series and the periodic boundary conditions are derived from the asymptotic analysis of the governing functional. This new micromechanical modeling approach has been successfully used to predict the effective thermoelastic properties including the elastic constants, specific heats, and coefficients of thermal expansions, and effective thermal conductivity and associated local fields [17–19]. It is also applied to accurately predict the initial yielding surface and elastoplastic behavior of metal matrix composites [20]. Since the procedure is quite similar, the authors have chosen to repeat some formulae and text from their previous publications in order to make the present paper more self-contained.

2. Magnetostrictive materials and its constitutive relation

The elastic and the magnetic responses are coupled in magnetostrictive composite materials where the mechanical variables of stress and strain are related to each other as well as to the magnetic variables of magnetic field and magnetic displacement. The coupling between mechanical and magnetic fields is described by piezomagnetic coefficients. Using the conventional indicial notation, the linear coupled constitutive equations can be expressed as:

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\epsilon_{kl} - q_{ijk}H_k, \\ B_i &= q_{ikl}\epsilon_{kl} + \mu_{ik}H_k \end{aligned} \tag{3}$$

where σ_{ij}, ϵ_{ij}, H_i, and B_i are the stress tensor, strain tensor, magnetic field vector, and the magnetic displacement vector, respectively. C_{ijkl} denotes fourth-order elasticity tensor at constant magnetic field, μ_{ij} is the second-order magnetic tensor at constant strain field, q_{ijk} is the third-order piezomagnetic coupling tensor.

Based on the representation of Li [6], Eq. (3) can be represented in a single constitutive relations as

$$\sum_{iJ} = E_{iJMn} Z_{Mn} \tag{4}$$

where

$$\sum_{iJ} = \begin{cases} \sigma_{ij} & J = 1,2,3 \\ B_i & J = 4 \end{cases} \tag{5}$$

$$E_{iJMn} = \begin{cases} C_{ijmn} & J,M = 1,2,3 \\ q_{nij} & J = 1,2,3; M = 4 \\ q_{imn} & J = 4; M = 1,2,3 \\ -\mu_{in} & J,M = 4 \end{cases} \tag{6}$$

$$Z_{Mn} = \begin{cases} \varepsilon_{mn} & M = 1,2,3 \\ -H_n & M = 4 \end{cases}. \tag{7}$$