Composite Structures 106 (2013) 537–548

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Interlaminar stress calculation in composite and sandwich plates in NURBS Isogeometric finite element analysis

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ARTICLE INFO

Article history:
Available online 10 June 2013

Keywords:
NURBS Isogeometric analysis
Laminated composite plate
First-order shear deformation theory
Interlaminar stress calculation
Direct integration
k-Refined NURBS higher order element

ABSTRACT

This research paper describes the development of NURBS Isogeometric finite element analysis and post-processor for interlaminar stress calculation in composite and sandwich plates. First-order, shear-deformable laminate composite plate theory is utilized in deriving the governing equations using a variational formulation. Linear, quadratic, higher order and *k*-refined NURBS elements are constructed and numerical validation is performed for laminated composite and sandwich plates. Lagrange finite element suffers from higher order stress gradient oscillations due to Gibbs phenomenon and require alternative stress recovery procedures for accurate interlaminar stress calculations, especially interlaminar normal stress. In this paper, direct post-processing is performed which computes interlaminar shear and normal stresses from higher order gradients of NURBS basis in a single step procedure. Interlaminar stresses are found to be in an excellent agreement with 3D elasticity solution and FSDT along with *k*-refinement procedure of NURBS basis is found to compute equivalent or better interlaminar normal stress than higher-order shear deformation theory.

1. Introduction

Inspite of extensive use of finite element methods, the barriers between engineering design and analysis still exist and the way to bridge gap is to reconstitute the entire process. Idea is to use one model and use it as an analysis model which require a change from classical finite element to an analysis procedure based CAD representation, formally known as Isogeometric analysis. There are several CAD functions which can be used for CAD representation of analysis module. Of most widely used CAD basis in engineering design process are NURBS (non-uniform rational B-splines) as presented by Piegle and Tiller [1], Farin [2], Cohen et al. [3] and Rogers [4]. NURBS basis are useful for analysis purposes because they possess useful mathematical property of refinement through knot insertion and variational diminishing property of convex hull. There are other computational geometry technologies that can be utilized as the basis for Isogeometric analysis such as sub-division surface by Peters and Reif [5] and Warren and Weimer [6] Gordon patches [7], Gregory patch [8], S-patch [9] and A patch [10], etc. Hughes et al. [11] introduced the idea of Isogeometric analysis using NURBS (non-uniform rational B-spline). They used NURBS to exactly represent the CAD geometry and then, constructed a coarse mesh for the analysis. The idea behind Isogeometric analysis is to model the geometry exactly which also serves the basis for the solution space i.e. invoking isoparametric concept. Kagan et al. [12] developed B-Spline finite element analysis and integrated with geometric design. Same authors [13] included adaptive refinement like *hp* and *h*-refinement techniques in their finite element code.

Advance multi-layered composite and sandwich plate/shell structures are being increasingly used in aerospace, shipbuilding, bridges and other industries. These structures have smaller thickness as compared to other dimensions and therefore, are often subjected to large deformation behavior under external loads. The plate theories for laminated composites can be categorized into equivalent single-layer theories (ELS) and layerwise theories. Equivalent single layer theories namely, classical, first-order and higher-order shear deformation theories, are derived from their 3D counterpart (i.e. layerwise and 3D elasticity) by making appropriate assumptions to the state of strain/stress in the thickness direction, reducing 3D continuum problem to a 2D problem.

Several articles are available in literature on shear deformation theories. Reissner–Mindlin theory [14–16] (first-order shear deformation theory, FSDT) assumes constant state of through the thickness strain. This theory was further extended for anisotropic plates by Whitney and Pagano [17]. Urthaler and Reddy [18] developed a mixed finite element for bending analysis of laminated composite plates using FSDT. They treated bending moment as a field variable

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http://dx.doi.org/10.1016/j.compstruct.2013.05.028

along with displacement and rotation. Similarly, Cen et al. [19], Kim et al. [20] and Mai-Duy et al. [21] have previously developed elements based on Mindlin–Reissner theory (FSDT).

In higher-order shear deformation theories (HSDT), Putcha and Reddy [22] developed a mixed finite element approach with 9-node Lagrangian quadrilateral element, Kant and Kommineni [23] formulated refined HSDT with 9-node quadrilateral element for linear and nonlinear finite element analysis of laminated composite and sandwich plates, Polit and Touratier [24] studied large deflection behavior using HSDT, Phan and Reddy [25] developed special third order theory (STTR) and Ren and Hinton [26] developed a third order plate theory for the analysis of laminated composite plates. Reddy and Robbins [27]'s provide extensive literature review on ELS and layerwise theories for laminated composite plates. Carrera [28] provide detailed review of zig–zag theories for multi layered plates and shells. Comparing higher-order theories with FSDT, higher-order plate theories enhance the accuracy of the solution slightly but are computationally more expensive. On the other hand, FSDT has often been used due to its simplicity and provide best compromise between economy and accuracy in predicting the response of thin to moderately thick laminates [29,30]. Kapoor and Kapania [31] developed NURBS Isogeometric finite element approach to study geometrically nonlinear behavior of composite plates. In this analysis, they were able to remove locking in thin plates and also were able to formulate a stable hourglass behavior using modified NURBS elements in parent domain.

It is a well known fact that composites are prone to damage under low transverse loads due to comparatively low transverse modulus and have higher probability of failure. The failure or damage is generally, initiated due to a variety of failure modes like delamination, matrix cracking, fiber failure, etc.; delamination being the primary mode of failure. Delamination is initiated when the interlaminar stresses attain the maximum interfacial strength. Therefore, predicting through-the-thickness stresses accurately is essential. This requires calculating higher-order derivatives of in-plane stress accurately which in turn requires higher-order displacement derivatives.

First order shear deformation theory with the use of constitutive relation produces highly inaccurate interlaminar stresses exhibiting non-physical discontinuity at the ply interface of composite laminate. The accurate of interlaminar stresses using 3D equilibrium equations in the context of finite element analysis depends upon the computation of higher-order in-plane strain gradients, which is generally poor as Lagrange polynomial gradient oscillates. Reddy [32] calculated transverse shear stresses using derivative of in-plane stresses that were obtained by differentiating interpolation functions in finite element approximation. In order to accurately predict delamination, interlaminar normal stress is as important as the shear stress. The computation of interlaminar normal stress requires an additional derivative of the basis function. These higher-order derivatives are not obtained directly in the finite element code. Tessler [33] developed smoothing variational formulation which combined discrete least square and penalty constraints functional in a single variational form and recovered the stress gradients more accurately. Byun and Kapania [34] developed a post-processing technique to overcome this drawback. They interpolated the finite element displacement data using polynomial functions like Chebyshev and orthogonal polynomials in a global domain. Use of Chebyshev polynomials require nodal displacement data to be available at some specific points i.e. Chebyshev points and cannot interpolate boundary edge nodal data while orthogonal polynomials use the arbitrarily distributed data points but are more difficult to obtain. Lee and Lee [35] introduced the non-iterative post processing procedure for the recovery of transverse stresses. They followed the equilibrium based stress recovery method, using the one dimensional, least square finite element in the thickness direction.

Park and Kim [36] presented a predictor–corrector post-processing procedure for the accurate recovery of stresses and displacement in the multi-layered composite panels. The predictor only predicts the transverse shear stress while the corrector method enhances the accuracy of the displacement, in-plane and transverse normal stress in the thickness direction using the results of the predictor and finite element analysis. Park et al. [37], later, used the nonlinear predictor–corrector method to obtain the stresses and displacement in composite panels in geometrically nonlinear formulation. Noor et al. [38] developed a computational procedure to get the transverse shear stresses in multi-layered composite panels. They first used the super convergent recovery technique to evaluate the in-plane stresses and then, used the piecewise integration in the thickness direction to obtain the interlaminar stresses.

Matsunaga [39] analyzed the displacement and stresses in composite laminated beams using global higher-order beam theory. Author expanded the displacement field variables with power series of z-coordinate. Makeev and Armanios [40] presented an iterative method to approximate analytical solution of elasticity problems in composite laminates. Rolfes and Rohwer [41] developed a method for calculating the improved transverse shear stresses in laminated composites using first-order shear-deformation theory. Kant and Manjunatha [42] developed numerical algorithms for accurate calculations of transverse stresses using higher-order shear-deformation theories. They used direct integration, finite difference and exact surface fitting approach. Recently, Kant et al. [43] proposed a semi analytical model for the accurate estimation of stresses and displacement in composite and sandwich structures. The two-point boundary value problem governed by a set of linear first-order differential equations through the thickness is solved using fourth-order Runge–Kutta method. Nosier and Bahrami [44] developed the analytical solution to study the edge effects in anti-symmetric angle-ply laminates using first order shear deformation and Reddy's layerwise theory.

Stress recovery is highly dependent on the structure of a particular element and on the formulation used in deriving elements. Stress recovery methods include interpolation-extrapolation from super-convergent points [46], L_2 projection [47], stress smoothing [48] and integral stress techniques [49]. Zienkeiwicz and Zhu [50] developed an efficient post-processing technique in terms of super-convergent patch recovery (SPR) procedure. A modified version of this technique was developed to obtain in-plane stresses at nodes and interlaminar stresses using equilibrium equations [52]. Stress recovery procedures suffer from extraneous stress oscillations [53].

This research present development of NURBS Isogeometric finite element and interlaminar stress calculations in laminated composite plates under sinusoidal load. Geometry is modeled exactly in Isogeometric framework and isoparametric and sub-parametric finite element representation is invoked for solution space. First-order, shear-deformable laminated composite plate theory is utilized in deriving the governing equations in variational formulation. Linear, quadratic, p and k-refined NURBS elements in parent domain are constructed. Interlaminar stresses are computed by direct integration of 3D equilibrium equations, utilizing non-oscillatory nature of higher order NURBS basis and their gradients. In-plane stresses are computed using constitutive equations and strain gradients for interlaminar stress computations are computed from higher-order gradients of NURBS basis. Most accurate interlaminar stresses, equivalent to 3D elasticity solution, are obtained by k-refined NURBS elements with coarsest meshes.