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A nonlinear finite element model using a unified formulation for dynamic analysis of multilayer composite plate embedded with SMA wires

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ABSTRACT

In this study, a new nonlinear finite element model is presented in the frame work of Carrera's Unified Formulation (CUF) for the dynamic analysis of SMA hybrid composite considering the instantaneous phase transformation and material nonlinearity effects, for every point on the plate. The CUF unify many theories in a unified form which can be differed by the order of expansion and definition of the variables in the thickness direction. The Brinson's SMA constitutive equation is used to model the behavior of SMA wires. The governing equations are derived using the Reissner Mixed Variational Theorem (RMVT) in order to enforce the interlaminar continuity of transverse shear and normal stresses between two adjacent layers. A transient finite-element-based method beside an iterative incremental procedure is presented to study the dynamic response of multilayered composite plate embedded with SMA wires. A suppressed vibration of the plate is observed, which is due to the energy dissipation of SMA wires. The parametric effects like length-to-thickness ratio, plate aspect ratio and also the effect of different boundary conditions, upon the loss factors are investigated. Results show that as the length-to-thickness ratio and also the plate aspect ratio increases, the loss factor decreases.

1. Introduction

Vibration damping consists of a challenging task to increase fatigue life and comfort of advanced structures. The variation of the properties of the material composing the structure provides a valuable approach to this task. Only, very limited options are available for such changes in the material properties. The present work proposes the use of NiTi shape memory alloys for this purpose. Presenting an exhaustive mathematical model for these structures is suffering from the nonlinearity behavior which is due to the phase transformation. Therefore, in the most cases, the proposed models are accompanied with a lot of simplifying assumptions. In the study by Jafari and Ghiasvand [1] dynamic analysis of SMA beam under a moving load was investigated. In their study, the effect of hysteretic loops was modeled by substitution of an equivalent damping ratio in the equation of motion. The dynamic analysis of SMA beam was studied by Hashemi and Khadem [2]. They considered the effect of the phase transformation. But In their research they assumed that, the beam behaves like a one-degree-of freedom system. Zbiciak [3] investigated the response history of SMA beam under impulse loading. He employed the rheological scheme for modeling the behavior of SMA material. He assumed that the material properties of SMA are constant.

Recently, many considerations have been focused on the improvement in the properties of the composite structures by shape memory alloys. In the study by Rogers and Barker [4] SMA wires were used to control the frequency of a graphite/epoxy multilayered beam. Upon the heating of SMA wires, an axial force was generated in the beam because of the shape memory effect. They showed that, the fundamental frequency of the beam was increased significantly by utilizing 15% volume fraction of SMA wires. Baz et al. [5] demonstrated that the SMA wires embedded to composite beams have a capability to control their natural frequencies. The effect of pre-strain, and also the effect of temperature on the SMA wires, was taken to account in their study. The effect of SMA wires on the controlling of the buckling and frequency analysis of composite beam was studied by Baz et al. [6]. They found that the buckling load of a flexible composite beam was increased up to three times the uncontrolled beam. Epps and

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Chandra [7] implemented an experimental–analytical investigation on the composite beams embedded with SMA wires. They demonstrated that, as the volume fraction of SMA wires increases the natural frequency of SMA composite beams increases. However, it cannot be found any details on the damping properties of SMA wires. Khalili et al. [8] studied the nonlinear dynamic response of sandwich beam with SMA hybrid composite skins under impulse loading. In their research a new element was proposed using the high order sandwich panel theory. They investigate the influence of the SMA wires on the vibration suppression of sandwich beam considering the phase transformation effects. In the research conducted by Ostachowicz et al. [9], dynamic and buckling analysis of composite plates embedded with SMA wires was investigated. They showed that the SMA wires have a significant effect on the natural frequencies and the thermal buckling of these structures. Lee and Lee [10] studied the buckling and post-buckling analysis multilayered composite plates embedded with SMA wires. They showed that the critical load of the composite plates is increased by activation of the SMA wires. Cho and Rhee [11] presented the nonlinear finite element model for static analysis of shape memory alloy wire reinforced hybrid laminate composite shells. They used the Brison's constitutive equation based on the iterative method for modeling the behavior of SMA wires.

The use of multilayer composite structures has been continuously growing in the recent years. Multilayered composite structures are utilized in many components of automotive, aerospace, and transportation vehicles. In recent years, many theories devoted to analyze the multilayer composite structures. Kirchhoff [12] (CLT) and Reissner–Mindlin [13,14] (FSDT) plate theories are not suitable to analysis the multilayered composite structures [15]. Because they cannot satisfy the continuity of transverse stresses between two adjacent laminate. In addition, these theories are not able to fulfill the zig-zag maner of the displacement distribution along the thickness direction. These conditions are called C_z^0-requirements in Ref. [16]. In this regard, a lot of theories have been presented for modification the FSDT [17–23]. These theories are known as higher-order shear deformation theories (HSDT). Khalili et al. [24] modified high-order theory for sandwich panels HSAPT, by applying first-order shear deformation theory for face-sheets and used the improved HSPAT to study the free vibration and low velocity response of sandwich panels. A lot of finite-element models are proposed using the HSDT models [17–23,25]. It should be mentioned that, a closed-form solutions can be found only in some few cases, especially for the linear problems with specific boundary conditions [26]. Two-dimensional theories are divided to some categories, based on the unknown variables. If the displacement field is only unknown, the corresponding theories are known as classical models and the governing equations are derived using the Principle of Virtual Displacements (PVD). If the transverse stresses are also assumed as unknowns, the corresponding theories are known as mixed theories [27,28]. Carrera et al. [16,29–33] presented a unified formulation (UF) of multilayered theories, for both the PVD and RMVT formulations. This can be referred as Equivalent-Single-Layer (ESL), if the unknown variables are considered for the whole plate, or (LW), if the unknown variables are considered for each layer, individually.

In this study, the nonlinear dynamic analysis of composite multilayered plate embedded with SMA wires is investigated based on the Carrera's unified formulation. The instantaneous phase transformation effects are considered for all the points on the plate for the first time. The Brinson's SMA constitutive equation is used to model the pseudoelastic behavior of shape memory alloys wires. In the present study, the (RMVT) is utilized to derivation of the governing equations. The governing equations of motion and the kinetic relations of phase transformation are coupled with each other. Therefore, a transient finite-element-based method beside an iterative incremental procedure is presented to study the dynamic response of multilayered composite plate embedded with SMA wires. Finally, a new program code is written in MATLAB software in order to dynamic analysis of composite plate embedded with SMA wires.

2. Constitutive equation of the SMA wires

In this study, the constitutive equation of shape memory alloys has been proposed by Brinson [34] is utilized. This constitutive equation presents the relation between the stress (σ), strain (ε), temperature (T) and martensite fraction (ξ) as follows:

$$\sigma - \sigma_0 = E(\xi)(\varepsilon) - E(\xi_0)(\varepsilon_0) + \Omega(\xi)(\xi_s) - \Omega(\xi_0)(\xi_{s0}) + \theta(T - T_0) \tag{1}$$

where $E(\xi)$ and θ are the Young's modulus and the thermoelastic coefficient, respectively. The subscript *0* implies the initial conditions of the corresponding term. $E(\xi)$ can be expressed as follows [34]:

$$E(\xi) = E_A + \xi(E_M - E_A) \tag{2}$$

where E_M and E_A are the Young's modulus of the shape memory alloys in the martensite and austenite phases, respectively.

In addition, $\Omega(\xi)$ is the transformation tensor and can be written in terms of Young's modulus as follows:

$$\Omega(\xi) = -\varepsilon_L E(\xi) \tag{3}$$

where ε_L is the maximum strain that can be recovered completely. In the model of Brinson, the martensite volume fraction is separated into two parts as follows:

$$\xi = \xi_s + \xi_T \tag{4}$$

where ξ_s indicates the fraction of the martensite that is induced by stress and ξ_T indicates the fraction of the martensite that is induced by temperature. Kinetic relations of the phase transformation (see Fig. 1) are expressed as follows [34]:

For conversion to martensite:

For $T > M_s$ and $\sigma_s^{cr} + C_M(T - M_s) < \sigma < \sigma_f^{cr} + C_M(T - M_s)$

$$\begin{aligned} \xi_s &= \frac{1 - \xi_{s0}}{2} \cos\left(\frac{\pi}{\sigma_s^{cr} - \sigma_f^{cr}}\left(\sigma - \sigma_f^{cr} - C_M(T - M_s)\right)\right) + \frac{1 + \xi_{s0}}{2} \\ \xi_T &= \xi_{T0} - \frac{\xi_{T0}}{1 - \xi_{s0}}(\xi_s - \xi_{s0}) \end{aligned} \tag{5}$$

For $T < M_s$ and $\sigma_s^{cr} < \sigma < \sigma_f^{cr}$

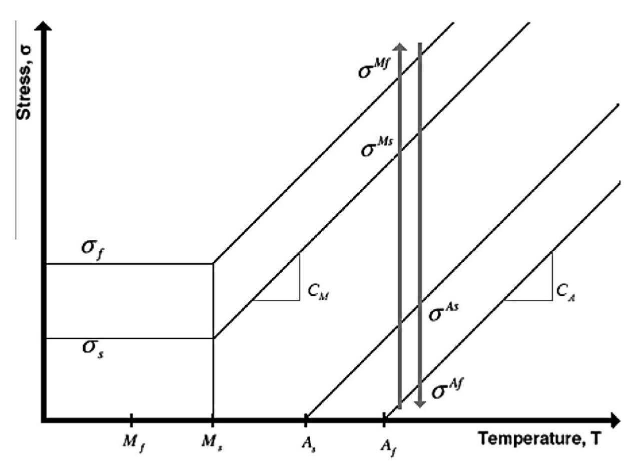

Fig. 1. Pseudoelastic behavior of shape memory alloys [35].