Composite Structures 106 (2013) 799–805

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Creep-rupturing of elliptical and circular cell honeycombs

Ting-Chun Lin, Ting-Jung Chen, Jong-Shin Huang *

Department of Civil Engineering, National Cheng Kung University, Tainan, 70101 Taiwan, ROC

ARTICLE INFO

Article history:
Available online 17 July 2013

Keywords:
Elliptical cell honeycombs
Creep rupturing
Power law creep
Modeling

ABSTRACT

This paper makes a theoretical analysis of the steady-state creep strain rates and creep rupturing times along the two principal directions of elliptical cell honeycombs using a unit cell model and assuming that solid cell walls follow power law creep and the Monkman–Grant relationship. Based on the results, the effects of the ellipticity of cell walls and relative density of elliptical cell honeycombs on their steady-state creep strain rates and creep-rupturing times can be evaluated. It is found that the Monkman–Grant parameters, m_1^* and m_2^*, of elliptical and circular cell honeycombs are equal to that of solid cell walls, m_s. In addition, the other Monkman–Grant parameters B_1^* and B_2^* decrease as the relative density increases, and B_2^* is always greater than B_1^*. Moreover, the creep strain rates and creep-rupturing times of elliptical and circular cell honeycombs are compared with those of regular hexagonal honeycombs with the same relative-density to evaluate the efficiency of their microstructures.

1. Introduction

The mechanical properties of hexagonal honeycombs depend on their relative densities as well as the geometry of their cell wall, and can be described by the cell-wall bending model presented in Gibson and Ashby [1]. That is, the solid distribution in cell walls plays an important role in determining the mechanical properties of hexagonal honeycombs. For example, the in-plane stiffness and strength of hexagonal honeycombs with variable-thickness cell walls can be higher than those with constant-thickness [2–5], and the out-of-plane elastic buckling strength of hexagonal honeycombs with curved cell walls tend to be significantly higher than those with straight ones [6]. Therefore, the mechanical properties of hexagonal honeycombs with a specific relative density can be improved by tailoring the solid distribution in their cell walls. Traditionally, elliptical and circular cell honeycombs with a carefully constructed simple cell-wall geometry have been widely used as lightweight load-bearing materials, and so the in-plane stiffness and strength of such cell honeycombs have been thus analyzed by many researchers [7–14], and the results indicate that they are superior to hexagonal honeycombs with straight and constant-thickness cell walls [13,14].

When hexagonal honeycombs are employed in lightweight structures at high temperatures, their creep behavior becomes critical, and should be taken into account to determine if they will be durable and safe during their service life. As steady-state creep proceeds at elevated temperatures, cell-wall creep-bending is common and cell-wall creep-rupturing eventually occurs when honeycombs are subjected to a uniaxial tension or compression. However, when honeycombs are under uniaxial compression at lower temperatures, there is another potential source of failure: creep-buckling of the slender, internal cell walls. The creep-rupturing and creep-buckling of hexagonal honeycombs with constant-thickness and straight cell walls can be analyzed theoretically using a cell-wall creep-bending model [15,16]. The time it takes for resulting creep-rupturing and creep-buckling of hexagonal honeycombs are related to their steady-state creep strain rates, and theoretical expressions for describing the creep-rupturing and creep-buckling of hexagonal honeycombs with curved and variable-thickness cell walls have been derived using the cell-wall creep-bending model [17–20]. It has been found that the creep behavior of hexagonal honeycombs is significantly affected by the solid distribution in cell walls, and their creep-rupturing time can be described by the well-known Monkman–Grant relationship [19]. Since cell-wall creep-bending is the dominant deformation mechanism when elliptical and circular cell honeycombs are uniaxially loaded at elevated temperatures, their creep-rupturing tendencies need to be known and understood and so should be investigated in detail. In this work, we use a unit cell model and first analyze the steady-state creep strain rates and creep rupturing times along the two principal directions of elliptical cell honeycombs subjected to a fixed uniaxial stress. Following this, the theoretical results of elliptical and circular cell honeycombs are compared with those of regular hexagonal honeycombs with straight and uniform-thickness cell walls to evaluate the efficiency of their microstructure.

* Corresponding author. Tel.: + 886 6 7943211; fax: +886 6 2358542.
E-mail address: jshuang@mail.ncku.edu.tw (J.-S. Huang).

0263-8223/$ - see front matter
http://dx.doi.org/10.1016/j.compstruct.2013.07.030

2. Theoretical analysis

Fig. 1 schematically illustrates an idealized elliptical cell honeycomb, with a unit thickness, a cell-wall thickness t_1, a major axis radius a and a minor axis radius b, subjected to a uniaxial tension or compression. The thickness of perfectly point-connected contact between any two adjacent elliptical cells is fixed and equal to $2t_1$, and the ellipticity of the cell walls, $\alpha = a/b - 1$, is defined as the degree of deviation of an ellipse from a circle. The coordinate of any position on each elliptical cell can be described by the set of a circumscribed circle with a radius a and an inscribed circle with a radius b, as shown in Fig. 2. The angle θ_1 measured counterclockwise from b axis can thus be geometrically calculated and expressed as $\theta_1 = \tan^{-1}[(1+\alpha)\tan\phi_1]$, while the angle θ_2 is measured clockwise from a axis is $\theta_2 = \tan^{-1}[\tan\phi_2/(1+\alpha)]$. A unit cell model surrounded with dashed lines in Fig. 1 is employed to analyze theoretically the steady-state creep strain rates along the two principal directions of the elliptical cell honeycomb. The relative density of the model honeycomb is the ratio of its density ρ^* to the density of solid cell walls ρ_s, and can be further expressed as: $\rho^*/\rho_s = \pi[(2+\alpha)(t_1/b) - (t_1/b)^2]/\left[2\sqrt{3}(1+\alpha)\right]$.

When the model honeycomb is subjected to a remote uniaxial stress σ_1^* along the x_1 direction, the induced forces P_1 and P_2 and moments M_a and M_b are exerted on the unit cell model, as shown in Fig. 3. From equilibrium, the following relation is found: $M_a = -M_b - P_2 b\sin(\pi/3) - P_1[(b - t_1/2) - b\sin(\pi/3)]$. An assumed uniform horizontal displacement δ_1 is shown in Fig. 3 and the stored elastic strain energy of the unit cell model U can be calculated from a static analysis. Since the model honeycomb is subjected to a remote uniaxial stress σ_1^*, the following constraints on the rotation and deformations of the unit cell model must be satisfied: $\partial U/\partial M_b = 0$ and $\partial U/\partial P_1 = \partial U/\partial P_2 = \delta_1$. By imposing the constraints with respect to P_1, P_2 and M_b on the stored elastic energy of the unit cell model [13,14], the induced forces and moments can be rewritten as follows: $P_1 = C_1 a\sigma_1^*$, $P_2 = C_2 a\sigma_1^*$, $M_a = C_a a^2\sigma_1^*$ and $M_b = C_b a^2\sigma_1^*$. Meanwhile, the coefficients C_1, C_2, C_a and C_b are expressed as:

$$C_1 = (-0.44\alpha^2 + 0.88\alpha + 1) \times \left[\frac{39.24 + 19.4(t_1/b) + (90.1 + 53.3\nu_s)(t_1/b)^2}{20.9 + 16.8(t_1/b) + (80.1 + 32.7\nu_s)(t_1/b)^2}\right] \quad (1)$$

$$C_2 = (-0.44\alpha^2 + 0.79\alpha + 1) \times \left[\frac{3.05 - 9.7(t_1/b) + (-60.7 - 3.3\nu_s)(t_1/b)^2}{20.9 + 16.8(t_1/b) + (80.1 + 32.7\nu_s)(t_1/b)^2}\right] \quad (2)$$

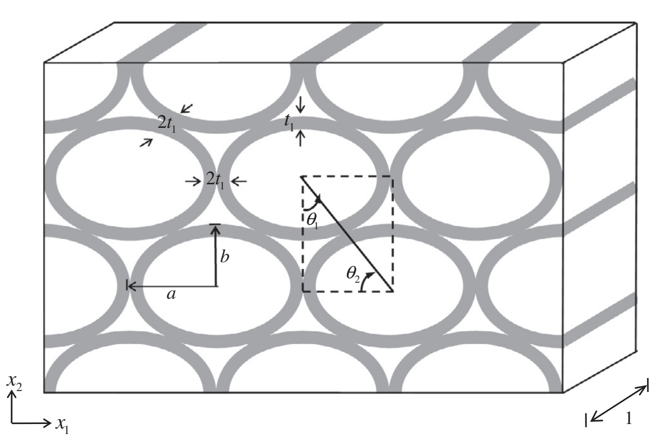

Fig. 1. A model elliptical cell honeycomb with a unit thickness, a cell-wall thickness t_1 and a major axis radius a and a minor axis radius b.

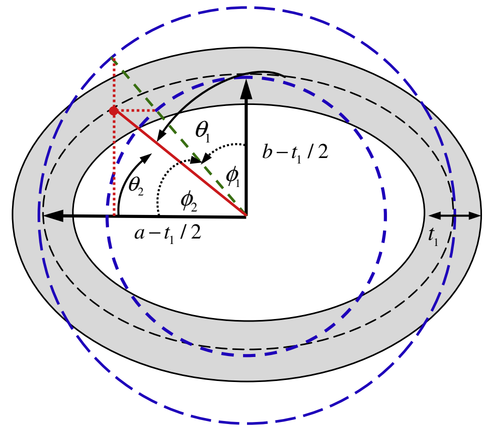

Fig. 2. The geometry of an ellipse can be described by a set of a circumscribed circle with a radius $a - t_1/2$ and an inscribed circle with a radius $b - t_1/2$.

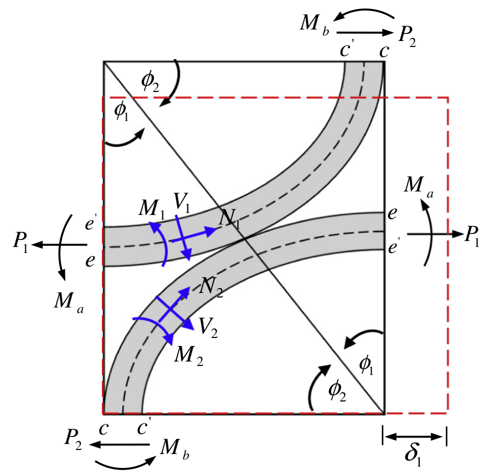

Fig. 3. The induced forces and moments exerted on the unit cell model for analyzing the creep-rupturing of the model honeycomb subjected to a uniaxial stress σ_1^* along the x_1 direction.

$$C_a = (-0.55\alpha^2 + 0.87\alpha + 1) \times \left[\frac{-62.3 + 176(t_1/b) + (-378 - 121\nu_s)(t_1/b)^2 + (397 + 333\nu_s)(t_1/b)^3}{394 + 316(t_1/b) + (1641 + 615\nu_s)(t_1/b)^2}\right] \quad (3)$$

$$C_b = (-0.49\alpha^2 + 0.64\alpha + 1) \times \left[\frac{6.46 - 6.46(t_1/b) + (-329 - 34\nu_s)(t_1/b)^2 + (226 + 85\nu_s)(t_1/b)^3}{197 + 158(t_1/b) + (821 + 308\nu_s)(t_1/b)^2}\right] \quad (4)$$

Here, ν_s is Poisson's ratio of solid cell walls. The bending moment M_1, axial force N_1 and shear force V_1 acting at any cross-section at an angle of ϕ_1 measured counterclockwise from the ee' sections, as shown in Fig. 3, can then be obtained from the equilibrium:

$$M_1 = a^2\sigma_1^*[-C_a - C_1(b/a - t_1/2a)(1 - \cos\phi_1)] \quad (5)$$

$$N_1 = a\sigma_1^* C_1 \cos\phi_1 \quad (6)$$

$$V_1 = a\sigma_1^* C_1 \sin\phi_1 \quad (7)$$

The bending moment M_2, axial force N_2 and shear force V_2 acting at any cross-section of the unit cell model at an angle of ϕ_2 measured clockwise from the cc' sections, as shown in Fig. 3, can also be determined from the equilibrium:

$$M_2 = a^2\sigma_1^*[C_b - C_2(b/a - t_1/2a)\sin\phi_2] \quad (8)$$

$$N_2 = a\sigma_1^* C_2 \sin\phi_2 \quad (9)$$

$$V_2 = a\sigma_1^* C_2 \cos\phi_2 \quad (10)$$