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Generalized differential quadrature finite element method for cracked composite structures of arbitrary shape

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ABSTRACT

This paper investigates the dynamic behavior of moderately thick composite plates of arbitrary shape using the Generalized Differential Quadrature Finite Element Method (GDQFEM), when geometric discontinuities through the thickness are present. In this study a five degrees of freedom structural model, which is also known as the First-order Shear Deformation Theory (FSDT), has been used. GDQFEM is an advanced version of the Generalized Differential Quadrature (GDQ) method which can discretize any derivative of a partial differential system of equations. When the structure under consideration shows an irregular shape, the GDQ method cannot be directly applied. On the contrary, GDQFEM can always be used by subdividing the whole domain into several sub-domains of irregular shape. Each irregular element is mapped on a parent regular domain where the standard GDQ procedure is carried out. The connections among all the GDQFEM elements are only enforced by inter-element compatibility conditions. The equations of motion are written in terms of displacements and solved starting from their strong formulation. The validity of the proposed numerical method is checked up by using Finite Element (FE) results. Comparisons in terms of natural frequencies and mode shapes for all the reported applications have been performed.

1. Introduction

Over the years, plate structures have been investigated by several researchers because plates are used as common structural components in civil, mechanical and aerospace engineering sectors. In particular, studies dealing with the dynamic behavior of flat plates are widely documented in literature [1–5]. Cracked rectangular isotropic plates have also been investigated and several convergence studies involving cracks and slits appeared in recent papers [6–9] and in papers published over the past decades [10–12]. Concerning the free vibration of laminated composite plates, the well-known First-order Shear Deformation Theory (FSDT) [13,14] is used. It should be underlined that relatively few published results are available for cracked plates with general boundary conditions. Over the years both the Finite Element Method (FEM) and the Ritz method have been used to work out numerical results [6–9,15–17]. Unlike the FEM, literature solutions do not consider the stress singularities at the crack tip. The Ritz method is suitable for solving vibration problems of simple geometry plates. It cannot be applied to arbitrarily shaped plates. In order to solve a problem with arbitrary boundary conditions and any kind of discontinuity through the thickness of the composite structure, a new numerical procedure named Generalized Differential Quadrature Finite Element Method (GDQFEM) is proposed in this work. This advanced numerical tool, which involves several subdomains of generic shapes, combines the strong formulation of a finite difference scheme and the general application of a finite element method. In this paper, the numerical implementation of the GDQFEM is used to analyze the free vibrations of arbitrarily shaped composite plates with cracks and slits [18–41]. To the best knowledge of the authors, no one has ever investigated the problem at issue. The GDQFEM is based on the Generalized Differential Quadrature (GDQ) method [18,42–77], which is a very fast and efficient methodology to solve systems of partial differential equations with associated boundary values. When the problem domain is rectangular, or more generally regular, the method can be easily applied to thick and thin plates, as well as to revolution shells, doubly curved shells, shells of translation and shallow shells [42–54,60–65,68–70,72–74]. It is underlined that, in practical applications, when the physical domain is usually complex, the GDQ method cannot be directly applied whereas the multi-domain technique can be used, also taking into account the presence of discontinuities.

As far as the domain decomposition is concerned, there are two basic approaches. The first method is called the multi-domain GDQ approach, in which the whole domain is simply decomposed into

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regular sub-domains. The second approach is based on a coordinate transformation procedure, where the generic shaped element in the actual coordinate system is transformed into a regular domain. The mapping approach follows exactly the same rules of the FEM. The only difference between them is that the transformation equations in the GDQFEM are based on the strong formulation, whereas the FEM is based on the weak form of the equations of motion.

2. Theoretical formulation

In this paper a First-order Shear Deformation Theory (FSDT) for plates has been implemented. This theory is also known as Reissner–Mindlin (RM) theory, due to the researches by Reissner [13] and Mindlin [14]. The main characteristic of this structural model is that it deals with a three dimensional problem of a plate as if it only were bi-dimensional. In fact, the plate geometry is described starting from its middle surface, which divides the plate thickness h into two equal parts. Since a composite laminate plate consisting of l layers is considered in the following, the total plate thickness can be expressed as

$$h = \sum_{k=1}^{l} h_k \tag{1}$$

where h_k is the thickness of the generic lamina k. The generalized application of FSDT to laminated plates has been extensively described by Reddy [5]. In this section, the main sets of equations which characterize laminated FSDT plates are summarized. In order to understand the merit and the limitation of the RM linear theory, the chief hypotheses on which the theory is based are examined. As it is well-known, the hypotheses at issue can be described by using the following restrictions on plates

1. the plate deflections are small and the strains are infinitesimal;
2. the transverse shear deformation is not negligible, and normal lines to the reference surface of the plate before deformation do not remain normal after deformation. The contribution of transverse shear stresses can be large;
3. the normal strain is assumed to be equal to zero: $\varepsilon_z(x, y, z) = 0$;
4. the plate is moderately thick, therefore it is possible to assume that the normal stress is negligible so that the plane assumption can be invoked: $\sigma_z(x, y, z) = 0$;
5. the linear elastic behavior of the material is supposed.

From numerical applications, it has been shown that the RM theory is in good agreement with the results obtained from 3D elasticity until the geometrical ratio h/L satisfies the following condition

$$\frac{1}{100} \leqslant \frac{h}{L} \leqslant \frac{1}{10} \tag{2}$$

where L indicates the shortest dimension of the plate under consideration. The FSDT considers five independent degrees of freedom. They are defined upon the middle surface of the plate and are constant through the thickness. The RM theory is also called the linear theory, because the displacements U and V are considered linear through the thickness, whereas the vertical displacement W is kept constant

$$\begin{aligned} U(x,y,z) &= u(x,y) + z\beta_x(x,y) \\ V(x,y,z) &= v(x,y) + z\beta_y(x,y) \\ W(x,y,z) &= w(x,y) \end{aligned} \tag{3}$$

It appears from Eq. (3) that the 3D displacements U, V and W are function of the 3D Cartesian coordinates, whereas the middle surface parameters u, v, w, β_x, β_y only depend on x and y coordinates.

From the RM displacement model (3), the three dimensional strain–displacement relationships are applied to obtain

$$\begin{aligned} \varepsilon_x &= \frac{\partial U}{\partial x} = \frac{\partial u}{\partial x} + z\frac{\partial \beta_x}{\partial x} = \varepsilon_x^0 + zk_x \\ \varepsilon_y &= \frac{\partial V}{\partial y} = \frac{\partial v}{\partial y} + z\frac{\partial \beta_y}{\partial y} = \varepsilon_y^0 + zk_y \\ \varepsilon_z &= \frac{\partial W}{\partial z} = \frac{\partial w}{\partial z} = 0 \\ \gamma_{yz} &= \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} = \beta_y + \frac{\partial w}{\partial y} = \gamma_y \\ \gamma_{xz} &= \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} = \beta_x + \frac{\partial w}{\partial x} = \gamma_x \\ \gamma_{xy} &= \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\frac{\partial \beta_x}{\partial y} + z\frac{\partial \beta_y}{\partial x} = \gamma_{xy}^0 + zk_{xy} \end{aligned} \tag{4}$$

The strain components derived in Eq. (4) link the strain characteristics ε_x^0, ε_y^0, γ_{xy}^0, k_x, k_y, k_{xy}, γ_x, γ_y to the middle plane displacement parameters u, v, w, β_x and β_y. It can be noticed that $\varepsilon_x^0, \varepsilon_y^0$ and γ_{xy}^0 are the in plane normal and shear deformations, respectively, k_x, k_y and k_{xy} are the plate curvatures, γ_x, γ_y are the out of plane shear deformations of the plate.

The relationship between stresses and strains is established through the constitutive equations, where the materials under consideration have linear elastic properties. In order to study thick laminated composite plates, a perfect bonding between layers is considered [5,71].

1. The bonding between layers is perfect (there is no flaw or gap between layers).
2. The bonding is non-shear-deformable (no lamina can slip relatively to another).
3. The bonding strength is as strong as it needs to be (the laminate behaves as a single lamina with special integrated properties).
4. The shear stresses at the top and the bottom of the plate are equal to the top and bottom external shear loading.

The last hypothesis derives from the fact that the starting 3D problem is turned into a bi-dimensional model, defined upon the middle surface of the plate. Using the hypotheses of the FSDT, the normal strain ε_n and the normal stress σ_n are negligible and only eight constitutive equations will be considered.

The stress resultants, which are defined as the integral of the corresponding stress components along the plate thickness, are related to the connected strain characteristics as follows

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ T_x \\ T_y \end{bmatrix} = \begin{bmatrix} A_{11}^{(0)} & A_{12}^{(0)} & A_{16}^{(0)} & A_{11}^{(1)} & A_{12}^{(1)} & A_{16}^{(1)} & 0 & 0 \\ A_{12}^{(0)} & A_{22}^{(0)} & A_{26}^{(0)} & A_{12}^{(1)} & A_{22}^{(1)} & A_{26}^{(1)} & 0 & 0 \\ A_{16}^{(0)} & A_{26}^{(0)} & A_{66}^{(0)} & A_{16}^{(1)} & A_{26}^{(1)} & A_{66}^{(1)} & 0 & 0 \\ A_{11}^{(1)} & A_{12}^{(1)} & A_{16}^{(1)} & A_{11}^{(2)} & A_{12}^{(2)} & A_{16}^{(2)} & 0 & 0 \\ A_{12}^{(1)} & A_{22}^{(1)} & A_{26}^{(1)} & A_{12}^{(2)} & A_{22}^{(2)} & A_{26}^{(2)} & 0 & 0 \\ A_{16}^{(1)} & A_{26}^{(1)} & A_{66}^{(1)} & A_{16}^{(2)} & A_{26}^{(2)} & A_{66}^{(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa A_{44}^{(0)} & \kappa A_{45}^{(0)} \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa A_{45}^{(0)} & \kappa A_{55}^{(0)} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \\ \gamma_x \\ \gamma_y \end{bmatrix} \tag{5}$$

The material stiffness parameters $A_{ij}^{(\tau)}$ depend on the elastic coefficients $\overline{Q}_{ij}^{(k)}$ and are defined as

$$A_{ij}^{(\tau)} = \sum_{k=1}^{l} \int_{\zeta_k}^{\zeta_{k+1}} \overline{Q}_{ij}^{(k)} \zeta^\tau d\zeta, \quad \tau = 0, 1, 2 \tag{6}$$