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C^0-continuous triangular plate element for laminated composite and sandwich plates using the {2,2} – Refined Zigzag Theory

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ABSTRACT

Most of the existing plate elements assume constant transverse displacement across the thickness resulting in zero transverse stretch deformation. This study presents a new triangular finite element for modeling thick laminates and sandwich panels based on the {2,2}-order refined zigzag plate theory. It adopts quadratic through-thickness variation of the in-plane and transverse displacement components. The transverse normal strain is calculated based on the assumption of cubic representation of the transverse normal stress. The zigzag functions are piecewise linear through the thickness. The element consists of 3 corner nodes and 3 mid-side nodes along the edges. Each corner and mid-side node has 11 and 3 degrees of freedom (DOF), respectively. This C^0 continuous element is free of geometric locking, and does not require shear correction factors. It provides robust and accurate prediction of all six stress components (in-plane and transverse normal and shear stresses) in the analysis of highly heterogeneous laminates and sandwich plates.

1. Introduction

A wide variety of modern civilian and military aircraft use fiber-reinforced laminates and sandwich panels for primary load-bearing structures. The use of adhesives (resin) is unavoidable when fabricating and constructing composite parts. As shown in Fig. 1, there commonly exist strong fiber-rich and weak resin-rich layers through the thickness; thus, failure can occur in multiple locations: core/face sheet interface, laminae interface and core. Component level structural testing and analysis is prohibitively expensive and time consuming. Instead, using robust and accurate computational tools complemented by experiments at key stages of the design is a viable and cost-effective option.

The stress state in a composite laminate or a sandwich panel is dependent on the loading conditions, fiber- and resin-rich layer thickness, face-sheet lay-up, core thickness and process-dependent properties. Understanding the behavior of such structures can result in weight and cost savings by designing against unnecessary conservatism. Finite element analysis (FEA) is widely utilized to assess the strength of such structures. However, an extremely detailed finite element discretization becomes necessary to model for all of the potential failure modes. Through-the-thickness finite element discretization using traditional elements is often impractical because it requires an extremely high mesh density to maintain a proper aspect ratio between the elements in the fiber-rich and resin-rich layers.

To evaluate the strength of a composite structure with a high degree of fidelity, it is first essential to predict accurate stress and strain fields, and then utilize a computationally efficient and accurate progressive damage model. For these reasons, a robust and accurate element that accounts for the discrete nature of fiber-rich and resin-rich layers of each ply as well as the variation of stiffness and strength properties of the core is necessary in the finite element analysis.

In order to achieve computationally robust and yet accurate finite element models for thick laminates as well as sandwich plates, investigators have focused on two types of elements, which are (1) the elements based on higher order plate theories and (2) the elements whose in-plane displacement filed is enhanced by zigzag theories. In elements derived based on higher order plate theories, the through-the-thickness variation of in-plane and transverse displacement fields of the plate are expanded in the form of quadratic or higher order polynomials or by employing trigonometric functions. These expansions enable the element to capture transverse shear deformations or both transverse shear and normal deformations with reasonable accuracy and computational cost. Reddy [1] presented a review of all the existing third-order theories and showed that they are indeed special cases of his third-order plate theory [2], in which the in-plane displacement components are cubic through-the-thickness expansions, yielding a quadratic

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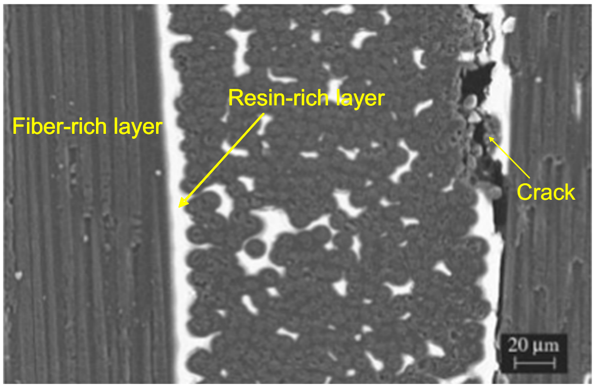

Fig. 1. Fiber-rich and resin rich layers in a laminate.

variation of transverse shear strains, and the transverse displacement component is constant through the thickness, excluding the transverse normal deformations. While this formulation is applicable to plates with simply supported boundary conditions [3], it yields physically unacceptable zero transverse-shear strains when both in-plane displacement components are fixed along the plate edges, as pointed out by Murty [4]. This shortcoming was remedied by decomposing the transverse displacement component into separate parts for the displacements associated with bending and shear deformations and has been applied successfully to stress [4,5], buckling [6], and large deflection analyses [1,7,8] of isotropic and laminated plates. An alternative to decomposing the transverse displacement field in order to avoid zero transverse shearing along fixed-edge boundaries was proposed by Voyiadjis and Shi [9] for thick cylindrical shells and later reduced to plate kinematics by Shi [10]. In this alternative approach, average displacement and slope variables were utilized that produce an equivalent transverse shear strain energy density. Similarly, Barut et al. [11] adopted the use of average displacements and slopes around plate boundaries by using Reissner's weighted-average displacement and slope definitions [12].

Other forms of higher-order theories for plates and shells were also proposed. For example, Soldatos and Timarci [13] and Timarci and Aydogdu [14] employed higher-order theories based on polynomial, trigonometric, hyperbolic, and exponential expansions of in-plane displacements through the thickness and compared their relative accuracy in stress and buckling analyses of plates and shells. Xiaoping and Liangxin [15] introduced a third-order shear deformation theory that satisfies continuity of in-plane displacements and transverse shear stresses between adjacent layers. However, their representation of the displacement variables resembles that of first-order shear deformation theory (FSDT). Recently, Ray [16] extended the zeroth-order shear deformation theory (ZSDT) of Shimpi [17] to perform vibration analysis of simply supported laminated composites. Because of the cubic and constant through-the-thickness expansions used for the representation of, respectively, the in-plane and transverse displacement components, the ZSDT of Shimpi and Ray is a special case of the form introduced by Timarci and Aydoglu [14].

Tessler [18] and Tessler and Saether [19] introduced a second-order shear-deformation theory that includes quadratic expansions of transverse shear deformations and a linear expansion of transverse normal deformations through the thickness; their formulation was later on extended to account for geometric nonlinearity by Barut et al. [20]. Similarly, Reddy [1] and Barut et al. [21] formulated third-order plate theories for thick laminates, taking into account both transverse normal and shear deformations, as well as cubic variation of in-plane deformations. Although extensive effort were devoted to the development of higher order plate theories in the past, they generally suffer from either incomplete representation of the stress fields or the fact that they require transverse shear correction factors particularly for composite materials.

Zigzag theories were developed particularly to increase the accuracy of stress fields in layered composite materials. The polynomial expansions of in-plane displacements defined across the entire thickness are enhanced by piecewise linear approximations (zigzag variations) in each layer. These additional zigzag terms provide more realistic representation of in-plane deformations in laminated composites as well as sandwich structures. The early development of zigzag theories [22–32] suffered from the undesirable vanishing condition of transverse shear deformations along clamped boundaries and C^1-continuity requirement in finite element approximations [27].

In order to eliminate these shortcomings, Tessler and his co-workers [33–38] recently introduced the Refined Zigzag Theory (RZT). In their work, the polynomial approximation is based on the FSDT and the zigzag functions are derived by taking into account the transverse shear stiffness of each ply. While this approach results in discontinuous variations of transverse shear stresses along ply interfaces, they provide more accurate values of transverse shear stresses at the ply level, and the transverse shear strains do not vanish along clamed boundaries. Furthermore, the strains are defined in term of first derivatives of the kinematic variables; thus, allowing C^0-continuous finite element implementation.

Although the previous RZT studies are robust in representing in-plane as well as transverse shear deformations without requiring any transverse shear correction factor, their kinematic representations do not account for transverse stretching along the thickness of plates. Recently, Barut et al. [39] extended the Refined Zigzag Theory (RZT) introduced by Tessler et al. [36] to determine all six stress and strain components. In the extended RZT, the in-plane displacements are expanded in the form of piecewise quadratic functions, while the transverse displacement component is expanded with a quadratic polynomial through the thickness of the plate. Hence, the extended theory is defined by the notation of $RZT^{\{2,2\}}$, in which the first and second superscripts denote the order of expansions used for the in-plane and transverse displacement components, respectively. Based on this notation, the previous zigzag theory by Tessler et al. [36] is described as $RZT^{\{1,0\}}$. The displacement assumptions in $RZT^{\{2,2\}}$ involve eleven kinematic variables. As in the development of {1,2}-order plate theory by Tessler [19] and {3,2}-order single-layer plate theory by Barut et al. [40], the extended RZT also independently assumes a cubic variation through the thickness for the transverse normal stress component.

This study presents a new C^0 continuous triangular plate finite element based on $RZT^{\{2,2\}}$. The element consists of 3 corner nodes and 3 mid-side nodes along the edges. Each corner and mid-side node has 11 and 3 degrees of freedom (DOF), respectively. The element referred to as $RZE^{\{2,2\}}$ employs anisoparametric shape functions; thus, it is free of geometric locking. The element does not require shear correction factor because $RZT^{\{2,2\}}$ assumes constant shear strain variation within each layer. As a result, it does not violate its expected parabolic distribution, and does not require shear correction factors. Also, the higher order kinematics in $RZE^{\{2,2\}}$ leads to improved in-plane, transverse shear and bending behavior. The zigzag functions developed by Tessler and his co-workers [33–38] include individual layer shear rigidity, and their slopes when summed through the thickness vanishes. The element permits highly detailed modeling of each ply (fiber- and resin-rich layers regardless of the number of plies in the face sheets) as well as the non-homogeneous modulus of the core. Unlike the existing finite elements for composite laminates and sandwich panels, this new element provides robust and accurate predictions of all six