Computers and Geotechnics 103 (2018) 1–12

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Stability analysis of irregular cavities using upper bound finite element limit analysis method

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ARTICLE INFO

Keywords:
Irregular cavity
Upper bound finite element limit analysis
Stability number
Failure mechanism

ABSTRACT

Underground cavities mainly exhibit irregular forms in nature. Due to the difficulties in describing the geometrical shape of irregular cavities and theoretically analyzing such cavities, irregular cavities are generally simplified as regular circular, elliptical or polygonal shapes in most studies. The purpose of this paper is to quantitatively describe the shape of irregular cavities using Fourier descriptors from inverse discrete Fourier transform (IDFT) theory and to translate the generation process of irregular cavity contours into a conversion between discrete frequency-domain and time-domain signals. Based on this approach, the stability analysis of irregular underground cavities is conducted using the upper bound finite element limit analysis (UB-FELA) method. The upper bound stability numbers ($\gamma_{cr}D/c$) and collapse mechanisms are presented for a series of irregular cavity depth-to-diameter ratios (H/D), internal friction angles (φ) and geometrical shape descriptors (D_2, D_3 and D_8). The obtained results demonstrate that the stability numbers increase with increasing φ and decrease with increasing H/D and descriptor values (D_2, D_3 and D_8). The failure mechanisms are significantly asymmetrical due to the contour fluctuations of the irregular cavity. Specifically, interlaced shear bands form between the adjacent fluctuations on the surface of the irregular cavity, which considerably differs from the failure mechanism of regular cavities. Local shear failure is the main failure form for the soil around the cavity. An obvious bottom bulge phenomenon is observed in the cavity at small internal friction angles.

1. Introduction

The natural cavities in karst areas are hydrological and geomorphic phenomena formed by the erosion of soluble rock and transformation of groundwater and surface water over long periods of time. The geometrical shape of the natural cavities is random to an extent and exhibits significant irregularities. Because of the difficulties in describing the geometrical shape of a natural irregular cavity, circular, rectangular or elliptical geometries are assumed in most related studies [1–5]. However, these simplifications cannot reflect the irregular characteristics of natural cavities. Therefore, an applicable irregular cavity generation method to quantitatively describe the geometrical characteristics of irregular cavities is of great significance in practical engineering and has important reference value for studies of natural cavities in karst areas.

There are different views on whether the effect of the shape should be considered in the stability analysis of openings. Abbo et al. [6] indicated that the shape of a tunnel has an important influence on its stability. Du Mingrui et al. [7] investigated the influence of the shape on the strength characteristics and failure mechanisms of sandstone samples containing single prefabricated square, rhombus and ellipse voids based on a uniaxial compression experiment and suggested that the bearing capacity of sandstone samples containing square voids is the lowest and that the bearing capacity of sandstone samples with diamond-shaped voids is the highest. Baus et al. [1] analyzed the stability of the footings underlying circular, square and rectangular voids and compared the load-displacement curves of the three footings, therein suggesting that the effect of the void shape on the ultimate bearing capacity of the footing can be ignored. However, the given load-displacement curves in their paper only corresponded to a single void depth. And the influence of voids shape depends on the void depth to a certain extent. Based on the above analysis, the shape of the openings has a considerable influence on their stability. Despite the importance of this issue, previous studies of the influence of the shape of openings on the associated stability yielded insufficient results.

The intrinsic properties (shape, roughness, irregularity and many other factors) of a natural cavity considerably affect its overall stability and must be carefully considered to obtain reliable results. Mollon et al.

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https://doi.org/10.1016/j.compgeo.2018.06.018
Received 20 January 2018; Received in revised form 26 June 2018; Accepted 29 June 2018
0266-352X/

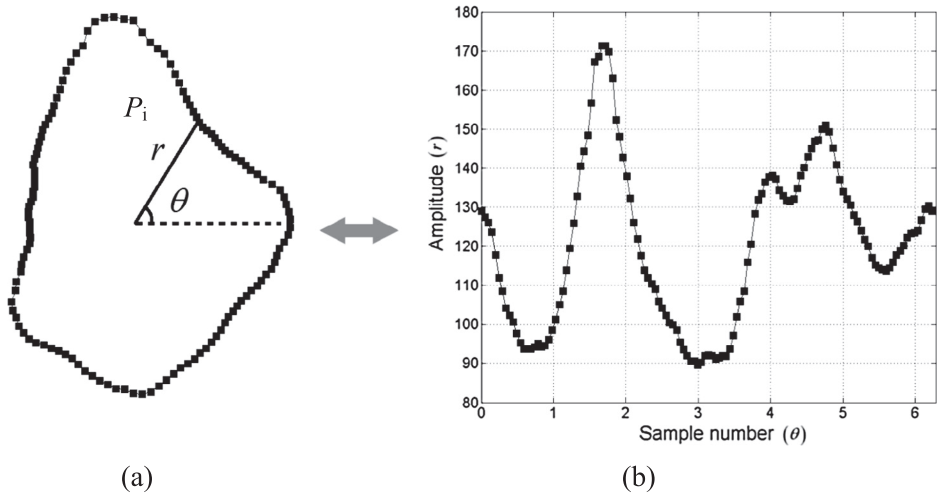

Fig. 1. The transformation process between the irregular cavity contour and discrete signal.

[8] proposed a novel method of generating realistic granules based on inverse discrete Fourier transform (IDFT) theory and accurately and quantitatively described the complex shapes of real particles with Fourier descriptors (D_2, D_3 and D_8). In addition, previous studies reported [8–10] that the Fourier descriptors in the frequency domain exhibit a strict correspondence with geometrical indices, such as the elongation and roughness of the irregular closed region. This relation indicates that the geometrical shape of an irregular cavity can be quantitatively determined. Based on these studies, this paper extends the above method to the generation of a natural irregular cavity and the shape of the irregular cavity is quantitatively characterized by Fourier descriptors in this process.

Previous studies have widely adopted finite element, limit analysis and finite element limit analysis methods to study the stability of underground spaces. Baus et al. [1], Badie et al. [2] and Wang et al. [5] analyzed the stability of the footings above circular and rectangular voids using the finite element method. Based on four types of shallow tunnel failure mechanisms. Davis et al. [11] investigated the stability of shallow buried circular tunnels under undrained conditions using the upper and lower bound limit analysis method and obtained the required supporting force in the ultimate limit state. Osman [12] studied the dual-tunnel stability in undrained clay using the upper bound limit analysis method. Unlike the finite element method, limit analysis ignores the stress-strain relationship of geotechnical materials and avoids the problem induced by the non-unique judgment standard of the limit state. The limit analysis method is an effective and direct method of analyzing the stability of geotechnical structures in their ultimate limit state [13,14]. However, under conditions with complex model boundaries and complicated strata, such as stability analyses of irregular cavities investigated in this paper, it is difficult to construct an appropriate failure mode, which implies that the accuracy of the traditional upper bound limit analysis method will be poor and cannot be used. The combination of the limit analysis and finite element methods can be used to solve the discrimination problem of the limit state of geotechnical structures and overcome the limitation of the presupposed failure mode [15]. Sloan et al. [16] were the first to apply the finite element limit analysis method in an undrained stability analysis of circular tunnels. Based on that result, Wilson et al. [17,18] investigated the stability of wide rectangular and dual circular parallel tunnels under undrained conditions using the finite element limit analysis method. Yamamoto et al. [19] studied the stability of a circular tunnel in cohesive-frictional soil subjected to surcharge loading using the finite element limit analysis method. As for 3D cavity research, Augarde et al. [20] researched the collapse of a spherical cavity using the 3D finite element limit analysis method. Unlike the stability analysis of the cavity under the condition of the plane strain condition, the realistic stress state of the soil mass can be reflected more comprehensively by using 3D model.

Based on the above analysis, the characteristics of the geometrical shapes of irregular cavities generated based on IDFT are described by Fourier descriptors (D_2, D_3 and D_8). The stability analysis of irregular cavity using the upper bound finite element limit analysis method is conducted in OPTUMG2 [21] (version 2018.02.27) with academic license. The obtained results can be used to estimate the potential collapse surface and scope of the collapse mechanism, which can then be reinforced in a timely and efficient manner. The shape of the irregular cavities is considered in the stability analysis, which is of great significance for reducing the damage produced by the collapse of natural irregular cavities, such as landscape cavities, and improving the construction safety in karst areas.

2. Generation and characterization of irregular cavities

Based on the characteristics of the mutual transformation between the time-domain and frequency-domain signals, the contour of the cavity is discretized as a time-domain signal, which indicates that the generation of the contour of an irregular cavity in the 2D case is converted in the reconstruction process of the time-domain signal in the field of signal processing. In addition, in this process, the frequency-domain signal can be used to precisely control the time-domain signal; thus, the geometric characteristics of the generated irregular cavity can be artificially controlled. The process of the transformation between the contour of an irregular cavity and the discrete signal is shown in Fig. 1 [10].

A proper center O (x_0, y_0) of the irregular cavity is chosen. The contour of the irregular cavity can then be characterized by N_p points P_i separated by a given angle θ_p with respect to O (for example, $\theta_p = 2\pi/N_p$; see Fig. 1(a)). To facilitate the calculation, the number of the points N (see Fig. 1(b)) that the signal contains is generally equal to 2^x, ($x \in N^*$) in the Fourier transform and is set to 128 in this paper. Thus, there are 64 harmonics, and the contour of the irregular cavity consists of 128 points in total. An example of the generation of an irregular closed contour region with $N = 128$ points is presented in Fig. 1 [10].

In Fig. 1, each point P_i is characterized by an angle θ_i and a radial distance r_i. In addition, the discrete time-domain signal r_i can be regarded as a uniform sampling θ_i in the range of $[0, 2\pi]$. Thus,

$$r_i(\theta_i) = r_0 + \sum_{n=1}^{N/2} [A_n \cos(n\theta_i) + B_n \sin(n\theta_i)] \tag{2.1}$$