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Research Paper

Probabilistic design optimization and simplified geotechnical risk analysis for large open pit excavations

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ABSTRACT

A probabilistic geotechnical design optimization framework for large open pit excavations is proposed in the present work. An appropriate objective function is formulated in terms of normalized cost of excavation, while the overall slope angle constitutes the decision variable. Probability of slope's failure is computed via Monte Carlo simulations. Optimization curves identifying the optimum slope's angle are developed for different risk and volume factors. For illustrative purposes, a typical example of a surface lignite mine is analyzed and results are thoroughly discussed. The suggested methodology can be used as a complementary tool in the decision analysis process of similar projects.

1. Introduction

The choice of slope's angle in surface mining has a significant economic impact on the operations of a mining company. This is so because steeper slopes result in lower stripping ratios, i.e. lower amount of overburden (waste) material to be excavated. For instance, Hustrulid et al. [19] presented a case study where for a 150 m deep open pit mine of 45° overall angle, overburden volume to be excavated approximately doubled for every 10° flattening of the slope. Given that the excavation cost is typically a great portion of the overall mining economics, design of pit slopes is a trade-off between stability and financial return [36]. As a result, there is often a strong economic motivation to design working pit slopes near economically optimum angles, despite the fact that a noteworthy amount of slope displacements is often expected at such angles [6]. In any case, slope design in open pit mines has been traditionally based on the limit equilibrium method and on empirical safety factors [20,33,7]. Recently, the introduction of the so-called partial factors and the characteristic/design values of soil properties, through design frameworks such as the European EC7 or the US LRFD, have contributed into a more robust consideration of geotechnical uncertainty. Nevertheless, conventional design frameworks still do not offer a direct reliability assessment of mining slopes. The reason is that neither global nor partial safety factors actually reveal the probability of failure (or generally of inadequate performance) of a mining slope. In fact, generally in geotechnical engineering, a safety factor can even be misleading in regards to the corresponding probability of failure, due to the ambiguity and the non-linearity that often exists between them and the corresponding risk level [11]. On the other hand, direct probabilistic analysis, as a supplementary tool in the process of a pit slope's design, offers the framework to overcome the above shortcoming, and it can be used in combination with risk analysis, which is often an important aspect on large scale engineering projects [2], such as the ones in mining engineering.

Implementation of direct probabilistic analyses in slope stability is not new in geotechnical engineering. Several researchers in the past have applied well established methods of reliability analysis, such as the First Order Second Moment, the Point Estimate, and the Monte Carlo Simulation, in order to consider geotechnical uncertainty and variability, and include the probability of failure, together with the traditional safety factor, in the calculated outcome (see among many others, [40,42,41,44,29,10,9,14,17,43,1]). However, most of the above works, despite their significance and contribution in the field of probabilistic slope stability, usually do not suggest a framework for probabilistic optimization in terms of the slope angle of the excavation.

In the present work, a probabilistic methodology leading to a practical geotechnical risk analysis and design optimization framework is developed for typical slopes of large open-pit excavations. First, geotechnical uncertainties are addressed by modeling shear strength properties as random variables following pre-assigned probability distributions. The probability of failure P_F is then computed on the basis of the method of slices through Monte Carlo simulations, providing further insight next to the value of the deterministic safety factor.

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Ultimately, an objective function expressed in terms of normalized excavation cost is formulated and its minimum value is sought, considering the overall slope angle as the decision variable. This function may be used for the probabilistic optimization of the design process and as such, it greatly facilitates the use of reliability as a tool on decision support analysis, which is often of significant importance in the mining industry. For illustrative purposes, the suggested methodology is applied for geotechnical risk assessment and probabilistic design optimization of a typical surface lignite mine, where a deterministic safety factor of 1.2 is commonly accepted. Results, based on a wide range of examined slope angles, are thoroughly discussed. The highly non linear correlation between the deterministic safety factors and the corresponding probabilities of failure is demonstrated. Parametric studies in terms of different levels of failure consequences indicate the potentials of the suggested methodology within the context of a decision analysis framework. In the first part of the paper, the theoretical basis of the suggested methodology is thoroughly presented. Stability principles are first presented, followed by the suggested frame of probabilistic modeling, risk analysis and design optimization in terms of the overall pit angle. The illustrative example follows in the second part of the paper.

2. Probabilistic modeling and optimization framework

2.1. Stability considerations

Generally, in slope stability problems, safety factor SF is traditionally defined by the ratio:

$$SF = \frac{s}{\tau} \tag{1}$$

where s is the available shear strength of the soil and τ is the shear stress at equilibrium. Considering a Mohr–Coulomb failure criterion in terms of effective stresses and/or forces, the above equation can be written as:

$$SF = \frac{c' + (\sigma - u)\tan\varphi'}{\tau} = \frac{c'\Delta l + (N - u\Delta l)\tan\varphi'}{\tau\Delta l} \tag{2}$$

where φ′ and c′ are the effective friction angle and cohesion of the soil, respectively, σ and N are the total normal stress and force on the shear plane, u is the pore water pressure, and Δl is the length of the shear plane. In order to compute SF, a slip (failure) surface is arbitrarily assumed and static equilibrium equations are used to compute stresses and SF for the chosen surface. The process is repeated for a large number of slip surfaces in order to identify the most critical one (i.e. the one with the minimum SF). Up to three static equilibrium conditions may be used: forces' equilibrium in the horizontal and vertical direction, and moments' equilibrium about any point.

In the present work, stability is addressed using the method of slices (i.e. the entire “failing” soil mass is separated in vertical slices, and equations of equilibrium are solved for each slice) according to Spencer's method [35]. Spencer’s method accounts for the shear and normal inter-slice forces between the slices and satisfies all requirements for static equilibrium (both forces and moment equilibrium). The basic assumption is that all inter-slice forces are parallel and inclined at an angle θ with respect to the horizontal, which is not known in advance, but it is computed as part of the problem's solution. Fig. 1 illustrates a compound slip surface (subdivided into slices) on a typical open pit mine excavation and the relevant forces acting upon a single slice. Two equilibrium equations, that represent overall force and moment equilibrium for the entire soil mass, are solved for the unknown safety factor SF and angle θ. The force equilibrium equation is expressed as [13]:

$$\sum Q_i = 0 \tag{3}$$

where

$$Q_i = Z_i - Z_{i+1} \tag{4}$$

on which Q_i is the resultant of the inter-slice forces, and Z_i and Z_{i+1} are the inter-slice forces on the two vertical sides of the slice, respectively. Since Z_i, Z_{i+1} are parallel, Q_i is their scalar difference. Taking moments about an arbitrary point, say the origin of a Cartesian coordinate system ($x = y = 0$), the equation for moment equilibrium can then be expressed as following [13]:

$$\sum Q_i (x_{bi}\sin\theta - y_{Qi}\cos\theta) = 0 \tag{5}$$

where

$$y_{Qi} = y_{bi} + \frac{M_{oi}}{Q_i \sin\theta} \tag{6}$$

on which x_b and y_b are the coordinates of the center of the slice's base, y_{Qi} is the y coordinate of the point on the line of action of the force Q_i, directly above the center of the slice's base, and M_o is the moment due to any known forces about the center of the slice's base. In the absence of any external loads on the slope's surface, or reinforcement internal forces, or seismic loads, the moment M_o is zero and therefore it holds true that $y_{Qi} = y_{bi}$. Considering force equilibrium ($\Sigma F_{xi} = \Sigma F_{yi} = 0$) for individual slices parallel and normal to the base of the slice, the following expressions can be written:

$$\sum F_{xi} = 0 \Leftrightarrow S + F_{vi}\sin\beta + F_{hi}\cos\beta + Q_i\cos(\beta - \theta) = 0 \tag{7}$$

and

$$\sum F_{yi} = 0 \Leftrightarrow N + F_{vi}\cos\beta - F_{hi}\sin\beta - Q_i\sin(\beta - \theta) = 0 \tag{8}$$

where F_{vi} and F_{hi} represent all horizontal and vertical forces on the i^{th} slice. Combining the above equations with Eq. (2) and solving for Q_i, it results that:

$$Q_i = \frac{-F_{vi}\sin\beta - F_{hi}\cos\beta - c'\frac{\Delta l}{SF} + (F_{vi}\cos\beta - F_{hi}\sin\beta + u\Delta l)\frac{\tan\varphi'}{SF}}{\cos(\beta - \theta) + \left(\sin(\beta - \theta)\frac{\tan\varphi'}{SF}\right)} \tag{9}$$

Substituting (6) and (9) into (3) and (5), gives two equations with two unknowns, i.e. the safety factor SF and the inter-slice force inclination θ. Solving for these two using a trial-and-error procedure eventually provides with their final values.

2.2. Probabilistic assessment

Opposite to a deterministic analysis, in a probabilistic slope stability approach, sources of uncertainty are characterized and explicitly accounted in stability's assessment. To represent the previously described limit state of equilibrium (Eq. (1)), it is convenient to define a performance function by analogy with conventional safety factor, as Safety Ratio (SR). So, safety ratio with respect to slope stability is expressed by:

$$SR = \frac{s}{\tau} \tag{10}$$

This mathematical expression is similar to that of Eq. (1), but in contrast with the safety factor, the safety ratio is a function of uncertain parameters. Given that these parameters will be modeled as random variables, τ and s are functions of random variables, so they are random quantities, too. Consequently, the term SR is a random quantity, as well. The probability P_F that the event [SR] is smaller than one is given by the following expression:

$$P_F = P[SR < 1] \tag{11}$$

while its complement, called reliability, is given by:

$$P_F' = 1 - P_F = P[SR \geqslant 1] \tag{12}$$

It is noted that another frequently used probabilistic measure of safety is the reliability index β, which is defined as the number of standard deviations between the most likely value of safety ratio and