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Research Paper

A hybrid plastic flow rule for cyclically loaded clay

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ABSTRACT

The strength loss of clays subjected to seismic loading is a crucial factor promoting slope instability. This paper proposes a constitutive modeling strategy to simulate accurately the development of pore pressure in cyclically loaded clays, i.e. one major factor responsible for strength deterioration during earthquakes. For this purpose, an enhanced plastic flow rule is proposed within the bounding surface framework to represent the cyclic behavior of reconstituted and/or weakly structured clays. The verification of the model against experimental evidence shows that the proposed flow rule improves the accuracy with which sign and magnitude of the pore pressure increments are computed.

1. Introduction

Clay slopes have failed spectacularly as a result of large earthquakes. A notable example are the landslides that took place in Anchorage as a result of the Alaskan earthquake of 1964 [54], for which the strength loss of facie III of the Bootlegger Cove Formation (BCF) clays has been recognized as a critical factor for failure initiation [54,21,37,61]. Similar evidence of strength loss due to seismic shaking have frequently been reported in a variety of geotechnical contexts [36,7,10], thus emphasizing the importance of accurately quantifying the strength deterioration of natural clays for purposes of hazard prevention and/or mitigation.

A common idealization of the mechanical behavior of natural clays involves the interaction between structure degradation (e.g., alterations of bonds and fabric) and unstructured clay matrix (i.e., the main factor controlling the response of clays in their reconstituted state) [8,32]. For this reason, Burland [8], Rampello [47] and Callisto and Calabresi [9] used the response of reconstituted clays as a reference to interpret the mechanical behavior of natural materials tested in the laboratory. Similarly, numerous constitutive models aimed at simulating the engineering response of natural clays [20,35,67,59,48] are essentially enhancements of baseline relationships originally conceived for reconstituted clays [51,49,17]. As a consequence, an efficient representation of the intrinsic behavior of cyclically loaded clays can be regarded as a mandatory step prior to the assessment of strength loss in any type of naturally structured geomaterial.

Many constitutive modeling frameworks have been proposed to replicate important features of intrinsic clay behavior under cyclic loading. Noticeable examples include multisurface plasticity, which embraces the concept of kinematic hardening [45,38,39,57,18], subloading surface plasticity [25], and finally bounding surface plasticity [4,28,33,53]. This work will be focused on the last framework, due to its mathematical simplicity (e.g., it usually requires defining only one surface) and its significant success in representing the cyclic behavior of soils.

Bounding surface models commonly use plastic flow characteristics defined at an image stress point (i.e., a projection of the current stress state on the outer bounding surface), which hereafter will be referred to as *image stress flow rules*. While convenient, image stress flow rules may underestimate the magnitude of pore pressure build-up during cyclic loading [52,56]. The influence of this discrepancy can be significant, in that the low effective stress arising when large pore pressure accumulates, as well as the corresponding reduction in stiffness and strength, likely are significant factors for catastrophic failures. To mitigate these shortcomings, this work uses experimental evidence to propose a new hybrid flow rule, i.e. a plastic strain operator which explicitly depends on flow characteristics determined both at *image* and *current* stress state. Although the proposed flow rule can be incorporated into any bounding surface constitutive law, here the model developed by Seidalinov and Taiebat [53] is used as a platform to assess its performance. This choice is motivated by the successful application of the aforementioned model to several clays [53], and it will facilitate a straightforward discussion of the benefits emerging from the use of the enhanced plastic flow rule. To test the capacities of the proposed flow rule, the model has been verified against laboratory evidence available for Georgia kaolin [55] and BCF clay [68,19,69], i.e. two clays which have been tested under the effect of widely different stress histories prior to cyclic loading. Furthermore, to capture the nonlinearity of cyclically loaded soils at

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both small and large strain levels, the hybrid flow rule has been used in combination with two additional model components, namely: (i) a small-strain elastic law [5] accounting for hysteretic effects at small to medium cyclic strain levels (e.g., 10^{-5}–10^{-2}); and (ii) a new expression of plastic modulus able to compute accurately the plastic strains that may lead to either ratcheting (i.e., strain rates that grow with the number of loading cycles) or shakedown (i.e., vanishing plastic effects after a large number of cycles) [50,55,71].

2. Formulation of the Seidalinov and Taiebat model

This section summarizes the key features of the model proposed by Seidalinov and Taiebat [53], which will be used as a platform to incorporate the new model components detailed in Section 3. Since the intrinsic clay behavior is the focus of this work, the destructuration mechanism in the original formulation by Seidalinov and Taiebat [53] is not considered. For the sake of simplicity, the constitutive relations are discussed with reference to axisymmetric stress conditions. For this purpose, the model is defined in terms of mean effective stress, $p = (\sigma_a + 2\sigma_r)/3$, and deviatroic stress, $q = (\sigma_a - \sigma_r)$, thus using the volumetric strain, $\varepsilon_v = \varepsilon_a + 2\varepsilon_r$, and deviatoric strain, $\varepsilon_d = 2(\varepsilon_a - \varepsilon_r)/3$, as their work-conjugate counterparts. Subscripts a and r denote axial and radial components, while v and d denote volumetric and deviatoric terms, respectively. All the stress variables are regarded as effective stresses and a compression positive convention is used for both stress and strain measures. To facilitate the implementation in numerical codes, the model generalization for multiaxial stress conditions is provided in Appendix.

2.1. Bounding surface and image stress

The yield surface proposed by Dafalias et al. [17] is adopted as anisotropic bounding surface. The latter can be represented in triaxial stress space as a rotated and distorted ellipse (Fig. 1(a)) characterized by the following expression:

$$F = (\bar{q} - \bar{p}\alpha)^2 - (N^2 - \alpha^2)\bar{p}(p_0 - \bar{p}) \tag{1}$$

The size of the bounding surface is determined by p_0, which grows or shrinks in proportion to plastic volume change in the same way as the Modified Cam-Clay (MCC) model [49]. Another internal variable, α, governs the rotation and distortion of the bounding surface, and represents the degree of plastic anisotropy. While in a general three-dimensional context, α would be represented by a second-order tensor, reference to axisymmetric conditions allows to describe it as a scalar, in that its principal directions coincide with those of the effective stress tensor. The model constant N denotes the stress ratio (i.e., $\eta = q/p$) at the peak of the bounding surface. Note that Seidalinov and Taiebat [53] assumes equal N regardless of mode of shearing due to the applied incremental loading path, which is distinguished by the position of the image stress relative to the rotation axis α (i.e., triaxial compression if $\bar{q} \geqslant \bar{p}\alpha$ and triaxial extension if $\bar{q} < \bar{p}\alpha$). In contrast, this work assumes that the value of N depends on the mode of shearing (i.e., $N = N_c$ for triaxial compression and $N = N_e$ for triaxial extension, where N_c and N_e are model parameters).

The variables $\bar{p}$ and $\bar{q}$ in Eq. (1) refer to the image stress shown in Fig. 1(a). The image stress is determined by the radial mapping rule proposed by Dafalias [14]. As shown in Fig. 1(a), a projection center, (p_c, q_c), is used to map radially the current stress, (p,q), to $(\bar{p},\bar{q})$ on the bounding surface. Mathematically, this relation can be expressed as:

$$\bar{p} = p_c + b(p - p_c); \quad \bar{q} = q_c + b(q - q_c) \tag{2}$$

where b varies from 1 to ∞, with these two end-members being attained when the current stress coincides with either the image stress ($b = 1$) or the projection center ($b = \infty$). Following Dafalias [14], the loading direction at the current stress is assumed to be the gradient of the bounding surface at the image stress (i.e., $\boldsymbol{L}$ shown in Fig. 1(a)).

2.2. Plastic flow rule and plastic potential surface

Seidalinov and Taiebat [53] employs an image stress flow rule, and accordingly defines the gradient of the plastic potential at the image stress (i.e., $\boldsymbol{R}_i$ in Fig. 1(b)) as the direction of the plastic strain increment. A plastic potential $g = 0$ which is different from the bounding surface is used, thus resulting in an non-associative flow rule. The plastic potential can be expressed as:

$$g = (\bar{q} - \bar{p}\alpha)^2 - (M^2 - \alpha^2)p_g(p_g - \bar{p}) \tag{3}$$

where p_g is a dummy variable so that the plastic potential surface can pass through the image stress. The parameter M is the stress ratio at critical state and $M = M_c$ if $\bar{q} \geqslant \bar{p}\alpha$ and $M = M_e$ if $\bar{q} < \bar{p}\alpha$, where M_c and M_e denote the critical state stress ratio in triaxial compression and extension, respectively. The volumetric and deviatoric components of the plastic flow direction (i.e., R_v^i and R_d^i in Fig. 1(b), respectively) can be accordingly given by:

$$R_v^i = \frac{\partial g}{\partial \bar{p}} = \bar{p}(M^2 - \bar{\eta}^2); \quad R_d^i = \frac{\partial g}{\partial \bar{q}} = 2\bar{p}(\bar{\eta} - \alpha) \tag{4}$$

where $\bar{\eta}$ is the stress ratio defined at the image stress.

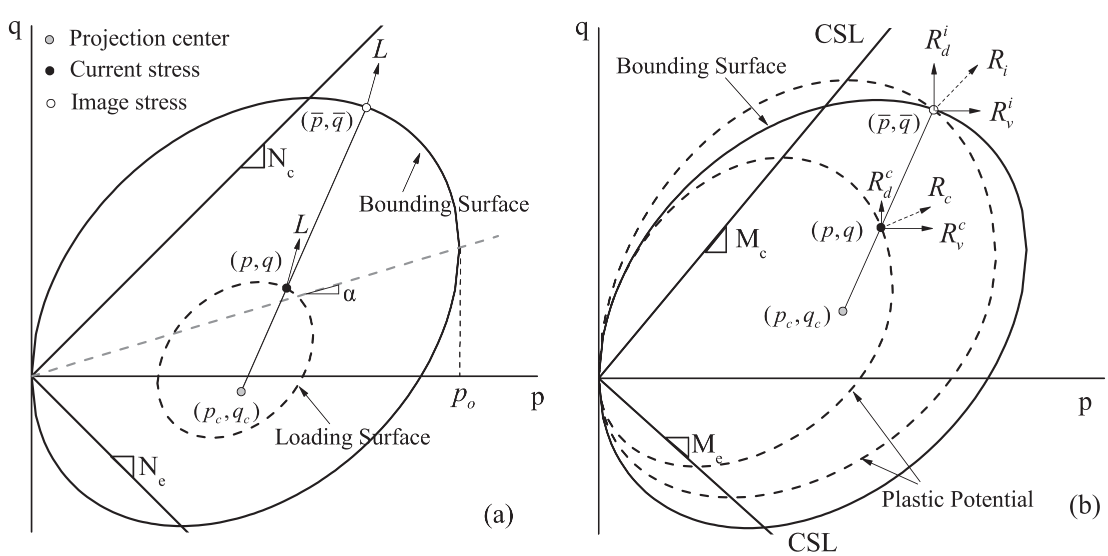

Fig. 1. Schematic diagrams of: (a) bounding surface and radial mapping; (b) plastic flow vectors at the current stress (R_c) and image stress (R_i).