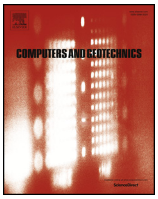
Computers and Geotechnics 101 (2018) 95–99

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo



Technical Communication

A new model for capturing void ratio-dependent unfrozen water characteristics curves

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ARTICLE INFO

Keywords:
Unfrozen water characteristics curve
Capillary water
Adsorbed water
Void ratio

ABSTRACT

A new unfrozen water characteristics curve (UWCC) model is proposed with explicit considerations of both capillarity and adsorption. Capillarity is considered to be dominant when the freezing of soil pore water occurs at higher temperatures, whereas the effects of adsorption are pronounced at lower temperatures. The proposed model is applied to predict UWCCs of silt at different initial void ratios and clay over a wide range of temperatures. It was found that the new model can capture the effects of void ratio on the UWCC. Moreover, the new model can better predict the UWCC over a wide range of temperatures.

1. Introduction

It is well recognized that the freezing of soil pore water takes place over a wide range of temperatures (i.e. from −50 to 0 °C). For solute-free soils, capillarity and adsorption are the two physical mechanisms controlling the entire freezing process of pore water [9]. The relationship between temperature and unfrozen water content in frozen soils is defined as the unfrozen water characteristics curve (UWCC). The UWCC is an important soil property for calculating frost heave and thawing settlement [21,15,24] as well as heat and water transport in frozen soils [14,13].

Until now, many semi-empirical models have been proposed for the UWCC. Anderson and Tice [1] found that the UWCC can be approximated as a power equation as follows:

$$\theta_w = \frac{r_d(1-\theta_s)}{100 r_w}\alpha(-T)^{\beta} \tag{1}$$

where θ_w is the unfrozen volumetric water content; T is the soil temperature (°C); α and β are soil parameters related to the specific surface area; r_d and r_w are the soil dry density and the water density, respectively; and θ_s is the saturated volumetric water content. It should be noted that Eq. (1) was formulated on the assumption that the adsorptive force governed the freezing of pore water. On the other hand, an exponential equation has also widely used for the UWCC as follows [23]:

$$\theta_w = \theta_{res} + (\theta_s - \theta_{res})\exp(\alpha_1 T) \tag{2}$$

where θ_{res} represents the residual volumetric water content, and α_1 is a soil parameter defining the freezing rate of the pore water. In this model, the freezing of residual water is ignored.

Alternatively, some UWCC models have been developed on the basis of soil water characteristics models and the Clausius-Clapeyron equation [8]. The soil water characteristics model proposed by van-Genuchten [19] has been widely used as follows [12,20]:

$$\theta_w = \left[1 + \left(\frac{L_w r_w \ln((T + 273.15)/273.15)}{\alpha_0}\right)^{n_0}\right]^{-m_0} \tag{3}$$

where α_0, n_0, and m_0 are soil parameters and L_w is the latent heat of fusion of water (333.7 kJ kg^{-1} at 0 °C). However, these semi-empirical models may not allow for a sound calculation for the UWCC over a wide range of temperatures, as they lack considerations of both capillarity and adsorption. Moreover, the effects of initial void ratio, which significantly influences the capillarity [7,2], on the UWCC are not explicitly considered in these models.

In this study, a new UWCC model is proposed, which separates the freezing of capillary and adsorbed water. An existing water characteristics model was extended for modelling the freezing of capillary water at different void ratios, and a new equation was proposed for the freezing of adsorbed water. Experimental results reported in the literature were used to verify the new model. The predictions using the new model were compared to those of Eqs. (1)–(3).

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https://doi.org/10.1016/j.compgeo.2018.04.019
Received 30 January 2018; Received in revised form 13 March 2018; Accepted 22 April 2018
0266-352X/

2. Development of a new UWCC model

2.1. Clausius-Clapeyron equation

In a closed system, the phase equilibrium between ice and unfrozen water can be described through the Clausius-Clapeyron equation. According to Koopmans and Miller [8], the Clausius-Clapeyron equation can be expressed as follows:

$$\frac{dP_w}{r_w}-\frac{dP_i}{r_i}=\frac{L_w}{T+273.15}dT \tag{4}$$

where P_w and P_i are the water and ice pressure, respectively; and r_i is the ice density. The value of L_w is assumed to be temperature-independent and the ice pressure is taken as the atmospheric pressure [9]. The difference between ice pressure and water pressure is defined as cryogenic suction [16]. As the ice phase is assumed to be at atmospheric pressure, cryogenic suction (s) is equal to the negative pore water pressure as follows:

$$ds=-dP_w \tag{5}$$

Under these assumptions, the integral form of Eq. (4) is as follows:

$$s=-r_wL_w\ln\frac{T+273.15}{273.15} \tag{6}$$

For Eq. (6), cryogenic suction increases with a decrease in soil temperature. Cryogenic suction reduces the potential of pore water [4] and, hence, the pore water can remain a liquid at a given negative temperature. Similar to unsaturated soils, capillarity and adsorption are two mechanisms governing cryogenic suction [9]. Capillarity occurs because of the presence of a curved ice-water interface. The adsorption of water is attributed to the presence of exchangeable cation hydration, mineral surface, or crystal interlayer surface hydration [17].

2.2. Freezing of capillary water

Capillarity is mainly influenced by the pore size distribution of the soil specimen. In this study, the initial void ratio is selected as a parameter to represent the average pore size of the soil specimen. Gallipoli et al. [6] proposed an equation to describe the relationship between volumetric water content and the suction of a soil specimen. The equation incorporates the effects of initial void ratio on the volumetric water content at different suctions as follows:

$$\theta_w=\left[\frac{1}{1+(m_0e^{m_1}s)^{m_2}}\right]^{m_3} \tag{7}$$

where e is the initial void ratio and m_0, m_1, m_2, and m_3 are soil parameters. It should be noted that the capillary and adsorbed water were not separated in Eq. (7). In this study, Eq. (7) is modified and adopted to represent the freezing of capillary water as follows:

$$\theta_{cw}=(\theta_s-\theta_{amax})\left[\frac{1}{1+(m_0e^{m_1}s)^{m_2}}\right]^{m_3} \tag{8}$$

where θ_{cw} is the volumetric capillary water content; θ_s is the saturated volumetric water content, and θ_{amax} is the maximum volumetric adsorbed water content. The freezing of capillary water could be achieved by substituting Eq. (6) into Eq. (8):

$$\begin{aligned}\theta_{cw}&=(\theta_s-\theta_{amax})\left[\frac{1}{1+(m_0e^{m_1}s)^{m_2}}\right]^{m_3}\\&=(\theta_s-\theta_{amax})\left[\frac{1}{1+(m_0e^{m_1}r_wL_w\ln((T+273.15)/273.15))^{m_2}}\right]^{m_3}\end{aligned} \tag{9}$$

Previous studies [19,18] suggested that the water content at a suction of 1500 kPa could be selected as the residual water content of a soil specimen. In this study, the maximum adsorbed water content is determined using the same method as that of the residual water content [10]. Based on Eq. (6), a suction of 1500 kPa is converted to a temperature of −2 °C. Therefore, the value of parameter θ_{amax} was determined as the unfrozen volumetric water content at a temperature of −2 °C.

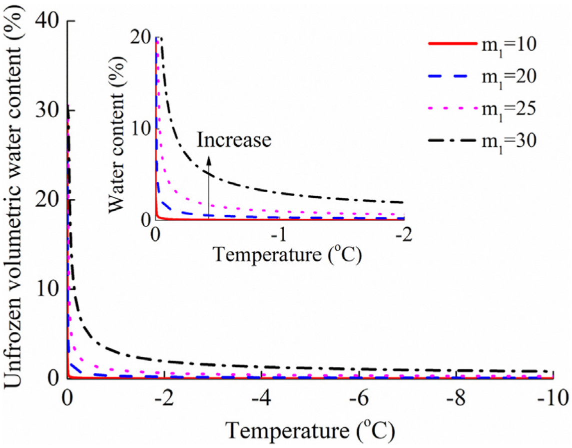

Fig. 1. Predicted freezing process of capillary water with different values of parameter m_1 in Eq. (9).

Comparing to the UWCC model derived from the v-G equation (see Eq. (3)), an extra parameter m_1 defining the influences of void ratio on the freezing of capillary water is added to Eq. (9). The physical meanings of parameters m_0, m_2, and m_3 are exactly the same as those in the v-G equation. Fig. 1 shows the effects of parameter m_1 on the freezing of capillary water. The parameters m_0, m_2, and m_3 are equal to 1500, 25, and 0.025, respectively. The values of e and θ_{amax} are equal to 0.7 and 10%, respectively. The value of m_1 ranges from 10 to 30. It can be seen that the unfrozen volumetric capillary water content decreases with a decrease in soil temperature for all four curves. The freezing rate of capillary water decreases with an increase in the value of parameter m_1. Moreover, during the phase change, the unfrozen volumetric capillary water content under a given temperature increases with an increase in the value of parameter m_1.

2.3. Freezing of adsorbed water

The freezing of pore water at low temperature ranges mainly occurs in adsorbed water [9]. Previous studies [3] have showed that the UWCC at low temperature ranges is in good agreement with the water characteristics curve as it converts the soil temperature to soil suction through the Clausius-Clapeyron equation. Lu [10] proposed an exponential equation to describe the water retention characteristics of absorbed water. In this study, a similar exponential equation is adopted for the freezing of adsorbed water as follows:

$$\theta_{aw}=\theta_{amax}\left\{1-\left[\exp\left(\frac{T-T_{\min}}{T}\right)\right]^k\right\} \tag{10}$$

where $T_{\min}$ is the temperature at which all the pore water is frozen and k is a soil parameter defining the freezing rate of adsorbed water. According to previous studies [5,11], there is a maximum suction at approximately 10^6 kPa corresponding to zero water content in any porous media. According to Eq. (6), the maximum matric suction of 10^6 kPa could be converted to a minimum temperature of −259 °C. Therefore, the parameter of $T_{\min}$ in Eq. (10) is determined to be −259 °C.

Fig. 2 shows the effects of parameter k on the freezing of adsorbed water. The value of θ_{amax} is 25%. The value of parameter k ranges from 0.01 to 0.08. The freezing rate decreases with an increase in the value of k. Furthermore, the freezing of absorbed water begins at approximately −2 °C with values of k ranging from 0.02 to 0.04. This is consistent with studies of unsaturated soils showing that the drying of