Computers and Geotechnics 101 (2018) 100–113

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Semi-analytical solutions to two-dimensional plane strain consolidation for unsaturated soil

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ARTICLE INFO

Keywords:
Semi-analytical solution
Unsaturated soil
Two-dimensional plane strain consolidation
Homogeneous and mixed boundaries
Finite sine transform
Laplace transform

ABSTRACT

This paper presents semi-analytical solutions to two-dimensional plane strain consolidation of unsaturated soils under different initial and boundary conditions. By applying the finite sine and Laplace transforms, the partial differential equations are converted to the ordinary differential equations. The semi-analytical solutions are obtained by using the finite sine inversion transform. Crump's method is adopted to perform the inverse Laplace transform to obtain analytical solutions in time domain. The present solutions are more general and can be degenerated into conventional solutions to one-dimensional consolidation of unsaturated soils. Several examples are provided to investigate two-dimensional plane strain consolidation behavior of unsaturated soils.

1. Introduction

In general, consolidation for soil is the process of the dissipation of excess pore pressures and the corresponding reduction in volume under long term static loads [1,2]. Since the inception of classical soil mechanics, Terzaghi's one-dimensional (1D) consolidation theory for saturated soils has formed an extremely useful conceptual framework in geotechnical engineering. As a case, vertical drains have been widely used to accelerate the consolidation process of fine-grained soils in preloading ground improvement projects. Barron [3] and Hansbo [4] have derived the consolidation solution for the sand drainage well, and then various analytical solutions were developed for consolidation of soft foundation by vertical drains [5–8]. However, sometimes the settlement cannot be estimated by existing consolidation theory very well in engineering practice. This is partly because the soil deposits above the phreatic line are unsaturated [2], and the solutions to the consolidation of unsaturated soils are more general. In addition, the consolidation behavior of unsaturated soils during vertical drain should be investigated, and it is essential to extent the concept of consolidation of unsaturated soils in two-dimensional (2D) problem [9].

As a common issue in geotechnical engineering, a considerable number of studies on the consolidation theory for unsaturated soil has been conducted, and a great progress has been achieved. Such as, Scott [10] estimated the consolidation of unsaturated soils with occluded air bubbles. Biot [11] proposed a general consolidation theory that is suitable for analyzing unsaturated soil with occluded air bubbles. Barden [12] presented an analysis of the 1D consolidation of compacted unsaturated clay. By assuming that the air and water phases are continuous, Fredlund and Hasan [13] proposed a 1D consolidation theory, in which two partial differential equations (PDEs) were employed to describe the dissipation processes of excess pore-air and pore-water pressures in unsaturated soils. This theory is now widely accepted. Dakshanamurthy and Fredlund [14] then developed 2D plane strain governing equations of unsaturated soils based on the heat diffusion concept for the two-dimensional condition. Later, Dakshanamurthy et al. [15] extended the 1D consolidation theory of unsaturated soils proposed by Fredlund and Hasan [13] to analyze three-dimensional (3D) case. By assuming all the soil parameters remain constant during consolidation, Fredlund et al. [2] presented a simplified form of the 1D consolidation equations for unsaturated soils.

Since the inceptions of 1D, 2D plane strain and axisymmetric consolidation theories of unsaturated soils, several analytical approaches have been frequently developed by Conte [16,17] Qin et al. [18,19], Zhou et al. [20], Shan et al. [21] and Ho et al. [22]. Conte [16] investigated coupled as well as uncoupled solutions about consolidation in unsaturated soil by using the Fourier transform, then Conte [17] extended this study and proposed a general formulation that can deal with coupled consolidation with plane strain as well as axial symmetry. Using the Laplace transformation and Cayley-Hamilton techniques, Qin et al. [18,19] presented analytical solutions to the 1D consolidation for a single-layer unsaturated soil subjected to instantaneous and exponentially time-dependent loadings, respectively. Zhou et al. [20]

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https://doi.org/10.1016/j.compgeo.2018.04.015
Received 9 November 2017; Received in revised form 16 March 2018; Accepted 16 April 2018

Nomenclature

a_a	changing rate of initial excess pore-air pressure along depth
a_w	changing rate of initial excess pore-water pressure along depth
b_a	initial excess pore-air pressure at top surface
b_w	initial excess pore-water pressure at top surface
C_a	interactive constant with respect to air phase
C_w	interactive constant with respect to water phase
$C_{v_x}^a$	coefficient of volume change with respect to air phase in x-direction
$C_{v_z}^a$	coefficient of volume change with respect to air phase in z-direction
$C_{v_x}^w$	coefficient of volume change with respect to water phase in x-direction
$C_{v_z}^w$	coefficient of volume change with respect to water phase in z-direction
g	gravitational acceleration
h	thickness of soil layer
k_{a_x}	coefficient of air permeability in x-direction
k_{a_z}	coefficient of air permeability in z-direction
k_{w_x}	coefficient of water permeability in x-direction
k_{w_z}	coefficient of water permeability in z-direction
l	length of soil layer
M	molecular mass of air
m_1^a	coefficient of air volume change with respect to a change in $\sigma - u_a$
m_2^a	coefficient of air volume change with respect to a change in $u_a - u_w$
m_1^s	coefficient of volume change with respect to a change in $\sigma - u_a$
m_2^s	coefficient of volume change with respect to a change in $u_a - u_w$
m_1^w	coefficient of water volume change with respect to a change in $\sigma - u_a$
m_2^w	coefficient of water volume change with respect to a change in $u_a - u_w$
n	natural number
n_0	initial porosity
q_0	external loading
R	universal gas constant
S_{r0}	initial degree of saturation
s	complex number frequency parameter
T	integral range of the finite sine and cosine transforms
t	time
u_a	excess pore-air pressure
u_{atm}	atmospheric pressure
u_a^0	initial excess pore-air pressure
u_w	excess pore-water pressure
u_w^0	initial excess pore-water pressure
w	settlement
$w*$	normalized settlement
x	investigated distance
z	investigated depth
γ_w	unit weight of water
ε_v	total volumetric strain
Θ	absolute temperature.

employed two alternative terms, ϕ_1 and ϕ_2, which consist of the excess pore-air and pore-water pressures, to convert the nonlinear inhomogeneous PDEs into traditional homogeneous PDEs, and then solutions for the single and double drainage conditions were obtained using the separation of variable method. Shan et al. [21] also deduced an exact solution to the 1D consolidation for unsaturated soils by adopting the separation of variable method. However, the final equations have been left undisclosed as the result of cumbersome derivation, and it is difficult to be followed by engineers. By adopting eigen-function expansion method and Laplace transform techniques, Ho et al. [22] discussed a simple yet precise analytical solution to the 1D consolidation of an unsaturated soil deposit under homogeneous boundary condition subjected to an instantaneous loading. These analytical solutions are for unsaturated soils with homogeneous boundary conditions that are permeable or impermeable to both air and water phases. In addition, Fredlund et al. [2] named a type of mixed boundary conditions, in which the boundary is permeable to air and impermeable to water at the top or bottom surface. For the mixed boundary conditions, Qin et al. [23] and Shan et al. [24] gave semi-analytical and analytical solutions by using their mathematical methods mentioned above, respectively. Furthermore, Zhou and Zhao [25] obtained a numerical solution to the 1D consolidation of unsaturated soils under various initial and boundary conditions, and complex time-dependent loadings by the differential quadrature method (DQM). However, these mentioned solutions are only used to predict the 1D consolidation problem. Afterwards, Ho and Fatahi [9,26] gave an analytical solution to the 2D plane strain consolidation of an unsaturated soil stratum under homogeneous boundary conditions subjected to instantaneous and time-dependent loadings using the same method as Ho et al. [22]. And as a further work, Ho and Fatahi [27,28] also gave an analytical solution to the axisymmetric consolidation of an unsaturated soil stratum under instantaneous and time-dependent loadings.

On the other hand, the dissipation of excess pore pressures is believed to be predominantly influenced by the lateral drainage during the 2D consolidation for unsaturated soils by vertical drains [9], and the dissipation of excess pore-air is easier than that of excess pore-water at the top or bottom boundary. That is to say, there is mixed boundary condition for the 2D consolidation for unsaturated soils, and the various initial and boundary conditions are more general and practical for consolidation problem of unsaturated soils by vertical drains. Therefore, as an attempt, this paper presents semi-analytical solutions to predict the dissipation of the excess pore-air and pore-water pressures, and settlement of unsaturated soil deposit using the 2D plane strain consolidation theory proposed by Dakshanamurthy and Fredlund [14] with the single, double and mixed drainage boundaries under different initial conditions. To obtain final solutions, the finite sine and Laplace transforms are used to convert the PDEs to the ordinary differential equations (ODEs), which are solved by applying the substitution method. It is found that the current solutions are more general and in a good agreement with the existing solutions in the literature, they also can be degenerated into that of 1D consolidation for unsaturated soils. Finally, examples are given to illustrate the 2D plane strain consolidation behavior of unsaturated soils. Changes are sufficiently demonstrated in the excess pore pressures and settlement under varying the parameter values of the ratios of air-water, horizontal-vertical permeability coefficients, depth and horizontal distance.

2. Mathematical model

2.1. Governing equations

Based on the 2D plane strain consolidation equations for unsaturated soils proposed by Dakshanamurthy and Fredlund [14], Ho et al. [9] proposed a referential profile of an unsaturated soil stratum with a finite depth h (in z-direction) and length l (in x-direction) under vertical loading q_0, as shown in Fig. 1. The soil system of interest