Computers and Geotechnics 101 (2018) 114–129

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Coupled effective stress analysis of insertion problems in geotechnics with the Particle Finite Element Method

Lluís Monforte[a,*], Marcos Arroyo[a], Josep Maria Carbonell[b], Antonio Gens[a]

[a] *Universitat Politècnica de Catalunya – BarcelonaTech, Department of Geotechnical Engineering, Campus Nord UPC, Gran Capità s/n, 08034 Barcelona, Spain*
[b] *International Center for Numerical Methods in Engineering (CIMNE), Campus Nord UPC, Gran Capità s/n, 08034 Barcelona, Spain*

ARTICLE INFO

Keywords:
Penetration test
Large strains
Particle Finite Element Method (PFEM)
Cone penetration test

ABSTRACT

This paper describes a computational framework for the numerical analysis of quasi-static soil-structure insertion problems in water saturated media. The Particle Finite Element Method is used to solve the linear momentum and mass balance equations at large strains. Solid-fluid interaction is described by a simplified Biot formulation using pore pressure and skeleton displacements as basic field variables. The robustness and accuracy of the proposal is numerically demonstrated presenting results from two benchmark examples. The first one addresses the consolidation of a circular footing on a poroelastic soil. The second one is a parametric analysis of the cone penetration test (CPTu) in a material described by a Cam-clay hyperelastic model, in which the influence of permeability and contact roughness on test results is assessed.

1. Introduction

Many activities in geotechnical engineering (probing, sampling, pile installation…) involve the insertion of a rigid body into the soil. In this kind of problem, large displacements and deformations of the soil mass always occur. The coupled hydro-mechanical response of the soil adds further complexity, even in cases where insertion speed is tightly controlled. Analysis of problems of rigid body insertion into soil masses had traditionally relied on highly idealized approaches such as geometrically simple cavity expansion mechanisms [60]. Although much insight is gained from such analyses, a number of basic features of the problem are left aside and, as a consequence, a host of not fully understood empirical corrections and methods have been relied upon for practical applications. Current interpretation of CPTu results [29,49,46] is a clear example.

Numerical simulation seems an obvious alternative to advance understanding in this area. However, the numerical simulation of rigid body insertion into soils is a complex task because the system exhibits many non-linearities, contact-related, material-related and also geometrical. The geometrical non-linearity was a fundamental obstacle to the Lagrangian formulations of the finite element method (FEM) that are successful in other areas of geotechnical engineering. Strong mesh distortion resulted in large inaccuracies and/or stopped calculation at relatively small displacements [15].

In the last decades several numerical frameworks have been developed to address those problems. Some approaches are not based on continuum mechanics and use instead discrete element methods [1,12]. Continuum-based approaches are however dominant, particularly for fine-grained soils. Within continuum-based methods the approach most frequently applied to geotechnical insertion problems has been that of Arbitrary Lagrangian-Eulerian formulations (ALE). ALE finite element formulations combine the Lagrangian and Eulerian kinematic descriptions, by separately considering material and computational mesh motions [17]. Several slightly different ALE methods have been applied in geomechanics; a comparative review was recently presented by Wang et al. [56].

A second continuum-based numerical framework is that of the Material Point Method (MPM). A set of particles (material points) move within a fixed finite element computational grid. Material points carry all the information (density, velocity, stress, strain, external loads…) which, at each step, is transferred to the grid to solve the mechanical problem. The computed solution allows updating of position and properties of the material points. Several implementations of MPM have been already used to model rigid body insertion into soils [52,10,11].

The Particle Finite Element Method (PFEM) is a third continuum-based approach that seems suitable to address geotechnical insertion problems. PFEM is actually an updated Lagrangian approach, but one that avoids mesh distortion problems by frequent remeshing. The nodes discretizing the analysis domain are treated as material particles the motion of which is tracked during the numerical solution. Remeshing in PFEM is based in Delaunay tessellations and uses low-order elements. PFEM was first developed to solve fluid-structure interaction problems

* Corresponding author.
E-mail addresses: lluis.monforte@upc.edu (L. Monforte), marcos.arroyo@upc.edu (M. Arroyo), cpuigbo@cimne.upc.edu (J.M. Carbonell), antonio.gens@upc.edu (A. Gens).

https://doi.org/10.1016/j.compgeo.2018.04.002
Received 7 December 2017; Received in revised form 19 February 2018; Accepted 2 April 2018
0266-352X/

[42] and then extended to other areas, like erosion, solid-solid interaction and thermo-plastic problems [41,47].

Within geomechanics, PFEM was initially applied to tool-rock interaction problems by Carbonell et al. [6,7]. Later, Salazar et al. [48], extended that code to include Bingham-like rheology to model flowslides. Zhang et al. [61,62] have also used PFEM in the context of soil flow problems.

G-PFEM is a PFEM-based code for the analysis of solid insertion problems in soils. G-PFEM has been implemented into Kratos [14], an object-oriented multi-disciplinary open-access platform for numerical analysis tool development. Previously [31], the authors have demonstrated the good performance of G-PFEM in total stress analysis. In Monforte et al. [33], the numerical stabilization techniques that underpin the method, both for the single phase and for two-phase cases, were presented in detail.

This work documents G-PFEM developments to model two-dimensional coupled hydromechanical problems for water-saturated soils in quasi-static conditions. Some initial developments along this line were briefly illustrated by Monforte et al. [32] and Gens et al. [19]. The paper is structured in two main sections. The first one presents the main features of the numerical method: governing equations, discretization, stabilization and mixed formulations, constitutive relations and the contact model. The second one illustrates the performance of the method in two reference problems: consolidation of a circular footing loading a poroelastic soil and CPTu insertion into a modified cam clay soil of varying permeability.

2. Numerical model

2.1. PFEM

PFEM is a mesh-based continuum method: the solution is computed in a finite element mesh built with well-shaped low order elements. This computational mesh evolves during problem solution by means of frequent remeshing. A cornerstone of the PFEM implementation used here is an efficient remeshing strategy [42]. Basic tasks used in that strategy include adaptive inclusion of new nodes, Delaunay tessellation based on nodes and element smoothing. A Lagrangian description of the continuum is used and information between meshes is transferred using interpolation algorithms. This general PFEM scheme is enriched with the inclusion of rigid bodies of specified motion that may contact, penetrate and reshape the discretized continuum.

Although it is not strictly necessary (e.g. [61]), low order finite elements are typically used in PFEM: linear triangles in two-dimensional models and linear tetrahedra in three dimensions. Linear interpolated elements have several advantages based on their simplicity: particles usually define exclusively the mesh nodes and no additional interpolations are needed after remeshing. The computational cost is also reduced with respect to high-order elements, even if stabilized mixed formulations are required.

The interpolation of state variables plays a crucial role in the accuracy of the results. To avoid excessive smoothing of internal variables, information is transferred from the previous Gauss points to the new ones. In this work, a nearest neighbor interpolation procedure is used; hence, new integration points inherit the information of the closer Gauss point of the previous mesh. This strategy ensures that information is maintained in elements that do not change during the meshing process. When new particles are inserted in the domain, variables are linearly interpolated from those of the previous mesh element. More details about remeshing and interpolation in PFEM can be found elsewhere [31,47].

PFEM has some commonalities with some ALE methods previously used in geomechanics, like the remeshing and interpolation technique by small strain (RITSS) [23] or the so-called efficient ALE approach (EALE) [39]; a discussion of similarities and differences with those techniques may be found in Monforte et al. [31].

2.2. Governing equations

We consider only water saturated soils. They are modeled as a two-phase continuum employing a finite deformation formulation. The equations of linear momentum and mass of the mixture are written following the movement of the solid skeleton, considering as unknown fields the solid skeleton displacements and fluid pressure. This is the $\boldsymbol{u}$-p_w formulation, an approximation of the generalized Biot equations valid at moderate velocities [63]. For pseudo-stationary cases, these equations may be expressed as [4,24]:

$$\begin{cases} \nabla\cdot\boldsymbol{\sigma} + \rho_m \boldsymbol{g} = \boldsymbol{0} & \text{in } \Omega_t \times (0,T) \\ \frac{\varphi}{K_w}\dot{p}_w + \nabla\cdot\boldsymbol{v} + \nabla\cdot\boldsymbol{v}^{\boldsymbol{d}} = 0 & \text{in } \Omega_t \times (0,T) \\ \boldsymbol{u}(\boldsymbol{X},t=0) = \boldsymbol{u}_0 & \text{in } \Omega_0 \\ p_w(\boldsymbol{X},t=0) = p_{w0} & \text{in } \Omega_0 \\ \boldsymbol{u}(X,t) = \overline{\boldsymbol{u}} & \text{in } \Gamma_{\overline{\boldsymbol{u}}} \times (0,T) \\ \boldsymbol{n}\cdot\boldsymbol{\sigma} = \boldsymbol{t} & \text{in } \Gamma_{\overline{\boldsymbol{t}}} \times (0,T) \\ p_w(X,t) = \overline{p}_w & \text{in } \Gamma_{\overline{p}} \times (0,T) \\ -\boldsymbol{n}\cdot\boldsymbol{v}^{\boldsymbol{d}} = j & \text{in } \Gamma_{\overline{j}} \times (0,T) \end{cases} \tag{1}$$

where $\boldsymbol{\sigma} = \boldsymbol{\sigma}' - p_w \boldsymbol{1}$ is the total Cauchy stress tensor, $\boldsymbol{\sigma}' = \widehat{\boldsymbol{\sigma}}'(\boldsymbol{F},V)$ is the effective Cauchy stress tensor, $\widehat{\boldsymbol{\sigma}}'$ stands for the appropriate constitutive equation for path dependent materials, $\boldsymbol{F}$ is the total deformation gradient whereas V represents the set of internal variables of the constitutive model. ρ_m is the spatial description of the soil density, defined as $\rho_m = (1-\varphi)\rho_s + \varphi\rho_w = \frac{\rho_m^0}{J} + \frac{J-1}{J}\rho_w$, ρ_s and ρ_w are the density of the solid and water phase respectively. φ is the porosity, whose variations changes in time due to deformation and it is actualized according to: $\varphi = 1 - \frac{1-\varphi_0}{J}$, where φ_0 is the initial state whereas $J = \det(\boldsymbol{F})$ is the Jacobian between the initial state and the deformed configuration. It is assumed that the solid phase is incompressible, whereas the water phase is almost incompressible, with bulk volume stiffness given by K_w.

A Large strain generalization of Darcy's law [8,24] is employed:

$$\boldsymbol{v}^d = -\boldsymbol{k}\cdot(\nabla p_w - \rho_w \boldsymbol{g}) \tag{2}$$

where $\boldsymbol{k}$ is the permeability tensor. When permeability is anisotropic it is advantageous to consider it constant in the material description and rotate it following the solid skeleton deformation [24]. Anisotropic and void-ratio dependent (Kozeny-Carman (Chapuis and Aubertin [9])) permeability definitions have been implemented in G-PFEM [20] but they are not considered further in here; all cases presented use a constant isotropic value of permeability, denoted k.

2.3. Weak form and discretization

The weak form of Eq. (1) is obtained following standard procedures [64], multiplying both field equations by a set of virtual displacements, $\boldsymbol{w}$, and virtual water pressure, q, integrating the equations over the deformed domain, Ω_t, and applying the divergence theorem:

$$\begin{cases} \int_{\Omega_t} \frac{\partial w_i}{\partial x_j}(\sigma'_{ji} - p_w\delta_{ij})\mathrm{d}\Omega_t = \int_{\Omega_t} w_i \rho_m b_i \mathrm{d}\Omega_t + \int_{\Gamma_{\overline{t}}} w_i t_i \mathrm{d}\gamma \\ \int_{\Omega_t} q\left(\frac{\dot{p}_w}{K_w} + \frac{\partial v_i}{\partial x_i}\right)\frac{1}{J}\mathrm{d}\Omega_t + \int_{\Omega_t} \frac{\partial q}{\partial x_i} v_i^d \frac{1}{J}\mathrm{d}\Omega_t = \int_{\Gamma_{\overline{j}}} q j \frac{1}{J}\mathrm{d}\gamma \end{cases} \tag{3}$$

Note that the integration of the mass balance equation takes place over the reference domain. This is not the only possibility and, for instance, Borja and Alarcón [4] integrate the mass balance equation directly over the current configuration (in other words, multiplying the equation by J) whereas Larsson and Larsson [24] formulate the mass balance in terms of fluid content (i.e., scaling the equation by $J\rho_w$).

After obtaining the weak form of the balance equations, the discrete equations of the hydromechanical formulation are obtained. First, let us introduce the interpolants: