Computers and Geotechnics 101 (2018) 130–140

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

A hybrid algorithm of partitioned finite element and interface element for dynamic contact problems with discontinuous deformation

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ARTICLE INFO

Keywords:
Finite element model
Localized discontinuous deformation
Rigid displacement
Contact problem
Dynamic analysis

ABSTRACT

This work introduces the interactive method of partitioned finite element and interface element (PFE/IE) to analyse the dynamic behaviours of structures with discontinuous deformations. The dynamic IE equations can be derived by combining the nonlinear equation based on the Newmark method and the dynamic equilibrium equation. The nodal displacement can be solved via PFE by combining the contact force into the total force vector; consequently, the failure state can be procured. PFE/IE improves the computational efficiency as nonlinear iteration is limited to the possible contact region. Dynamic results obtained with other techniques or experiments are introduced and compared to validate the accuracy and robustness of PFE/IE.

1. Introduction

Localized discontinuous fields in structures are very common in engineering. Internal contraction joints in the concrete dam, fractures in rock and/or concrete, faults in slopes and planes with structural weakness. The opening, closing and friction sliding of these discontinuous fields are subjected to earthquakes and could be attributed to strongly nonlinear dynamic contact problems, which are widely concerned for engineering safety [1–3]. In this regard, development of research on the dynamic contact behaviour of structures with the localized discontinuous field is vital and necessary. To provide reliable response data for structural analysis and optimization control, dynamic response of these systems needs to be calculated accurately.

Classical numerical methods can be grouped in two main categories of continuum based methods (e.g., FEM, FDM, BEM) and discontinuous based methods (e.g., DEM, DDA).

FEM (finite element method) is almost the most widely applied numerical method across the science and engineering fields. FEM development works support for solution of discontinuous deformation problems, including material heterogeneity, non-linear deformability, and complex boundary conditions. The thin layer element, interface element and spring element are developed [4–8]. FEM is handicapped by the requirement of small element size, conformable fracture path and element edges when simulating the process of fracture growth. However, it is still difficult to simulate the status conversion of open and close for the discontinuous deformation interface by the elastoplastic constitutive law. Although the nonlinear deformability occurs only in the localized region, the stiff matrix need to be updated in the whole computational domain. The solution is inefficient for the problem being taken as a nonlinear problem.

BEM (boundary element method) [9–11] is an alternative computational method for solving contact problems as the nonlinearity of the problem appears only on the boundaries of the contacting bodies. Since BEM only models boundaries of the solution domain, then it is very efficient in terms of computing time and complexity. However, BEM is limited when information is needed for many internal points. Moreover, for non-linear problems, such as elastoplastic and large strain analysis, BEM is also limited because the non-linearity no longer allows the partial differential equations to fully deform at the surface of the element volume and therefore some form of domain discretization is required.

DEM (distinct element method) [12–14] is tailored for problems with many discontinuities and large displacements, without the small deformation assumption of FEM. The calculation is relatively simple due to using explicit solutions to solve the balance equation and dispensing with solving the complicated large stiffness matrix. The spring element is used to describe the relative motion between different continuums in this method. However, the stiffness of the spring element is difficult to determine. The basic difference between DEM and continuum-based methods is that the contact patterns between components of the system are continuously changing with the deformation process for the former but are fixed for the latter. When the continuum is composed of a complex shape covering large areas, the modal composition does not reflect the change of the displacement in any place

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https://doi.org/10.1016/j.compgeo.2018.04.022
Received 8 December 2017; Received in revised form 14 April 2018; Accepted 23 April 2018
Available online 11 May 2018

accurately, especially the area near the stress concentration.

A representative of implicit DEMs is DDA (discontinuous deformation analysis), which originated in [15,16], and further developed in [17–20]. The DDA method takes blocks in continuum parts as constant strain elements, considering the rigid motion of the continuum at the same time. The displacements in the continuum are chosen as unknown variables, with the spring to describe the relationship between relative displacements and forces acting on the boundaries of the continuum blocks. Compared with explicit DEM, DDA method can converge with larger time step for dynamic problem and has a closed-form integration for stiff matrices of elements. Nevertheless, the mechanical parameters are difficult to determine for springs, the method cannot accurately simulate the transition from continuity to the local discontinuity.

The studies of dynamic problems are rather complex because of the non-homogeneity and nonlinearity. For dynamic problems involving continuity and discontinuity, it is much more difficult to take both spatial discretization error and time discretization error into account. The computational time increases much more than static ones. To adopt one numerical computation method singly is inappropriate.

Recently, the hybrid numerical method has been used in engineering, especially for dynamic analysis on stress/deformation problems involving continuity and discontinuity [21–24], such as fractured rocks, concrete cracking, slope sliding, etc. Jin et al. [25] presented a hybrid procedure of distinct boundary element for discrete rock dynamic analysis. Liu et al. [26] introduced a novel single-step integral method and extended to problems of the dynamic response of contactable interfaces in viscoelastic structures. Guyot et al. [27] coupled FEM and BEM for the study of the elastic contact problem with Coulomb friction and the stresses in contact. Chen et al. [28] studied the nonlinear contact model for seismic analysis of high arch dams based on the Parallel Finite Element Program Generator (PFEPG). Beyabanaki et al. [29] presented nodal-based 3D DDA by adding the finite element mesh into DDA to enhance the block's deformation capability. Zhang et al. [30] proposed interface stress element method (ISEM) based on Kawai's rigid body spring method to describe the discontinuous deformation of rock mass by dividing the structure into the discrete element and interface element. Du et al. [31] proposed a hybrid analytical and the implementation of the extended finite element method (XFEM) with the Fictitious Crack Model (FCM) method, which could address complex cracks and local refinement to analyse crack problems. Huang [32] applied the direct coupling of local discontinuous Galerkin (LDG) and natural boundary element method (NBEM) for a class of three-dimensional nonlinear-to-linear interface problems on an unbounded domain by introducing a spherical surface as an artificial boundary. Zheng et al. [33] descried DDA-d with the contact forces as the basic variables to satisfy the contact condition, as well as momentum conservation. Li et al. [34] proposed an interactive method of Interface Elements and partitioned finite elements (PFE/IE) to predict the static failure process of a structure with local discontinuity by solving equations of PFE and IE separately and requiring no spring for possible discontinuous surface when it is in a continuous state. Li et al. [35] proved the applicability of PFE/IE for stability analysis of local discontinuous rock mass in slope system.

So far as possible, PFE/IE is accurate and efficient when dealing with local discontinuity problems whose failure surfaces are apparent. The calculation efficiency of PFE/IE improves distinctly compared with continuum based methods. Otherwise, PFE/IE is more applicable to the Localized discontinuous systems, which are best approximated by a combination of large area continuum and small area discontinuum, rather than the discontinuous based methods. Hybrid methods usually use the coupled model which set up with somewhat different definitions of contact. Compared with the combinations of FEM, DEM, or DDA, there is no need to communicate information between the two solvers to ensure the smooth transfer of momentum and energy. Waste of computational resource is avoided, because there is no limit on time increment of PFE/IE method. However the coupling method must choose the smaller time increment to ensures numerical stability of the continuum and discontinuum based methods.

The aim of the present study is to explore the dynamic response of the structure coupling continuum and discontinuum and to resolve complex contact problems involving material fracturing, dynamic behaviour and contact effect. An improvement of a dynamic algorithm for simulating the structure failure process and searching the contact area between different discrete bodies in the hybrid PFE/IE framework is presented in this paper. The advantage of the proposed hybrid method in dynamic contact problems is validated through several tests, whose calculated results are in good agreement with analytical solution and experiment results supported by references.

2. Hybrid method for dynamic problems

2.1. Dynamic finite element equation of solid blocks

Assuming several discontinuous interfaces divide the whole structure into n solid blocks. For each node in the ith block, δ_i^g, $\dot{\delta}_i^g$, $\ddot{\delta}_i^g$ are rigid displacement, velocity and acceleration moving with the block's centroid respectively. δ_i, $\dot{\delta}_i$, $\ddot{\delta}_i$ are relative displacement, velocity and acceleration to the centroid respectively. $\delta_{t,i}$, $\dot{\delta}_{t,i}$, $\ddot{\delta}_{t,i}$ are the total displacement, velocity and acceleration respectively. The relationship between those variables are

$$\begin{aligned} \delta_{t,i} &= \delta_i^g + \delta_i \\ \dot{\delta}_{t,i} &= \dot{\delta}_i^g + \dot{\delta}_i \\ \ddot{\delta}_{t,i} &= \ddot{\delta}_i^g + \ddot{\delta}_i. \end{aligned} \quad (1)$$

The number of unknown variables that represent rigid displacement depends on constraint levels of the block.

To simulate the discontinuous deformation and transfer contact forces between diverse blocks, m groups of the discontinuous interfaces and $p_i\,(i = 1,2,...m)$ pairs of contact points on the ith discontinuous interface is assumed. In total, there are mp ($mp = \sum_{i=1}^{m} p_i$) pairs of contact points. The local coordinate system in each pair of contact points ($k = 1,2,...mp$) is expressed as $\xi_k\eta_k\varsigma_k$ (ς_k is the normal direction). T_k is transfer matrix from local to global coordinate. Take the constraint internal forces in contact points under the local coordinate system as $F_{cl} = \{f_{cl_1}\ \ f_{cl_2}\ \cdots\ f_{cl_k}\ \cdots\ f_{cl_{mp}}\}^T$. Then, take the constraint internal force imposed on the kth pair of contact points as $f_{cl_k} = \{f_{cl_{k\xi}}\ \ f_{cl_{k\eta}}\ \ f_{cl_{k\varsigma}}\}^T$. In result, the constraint internal forces under the global coordinate system will be

$$F_{cg} = \left\{f_{cg_1}\ \ f_{cg_2}\ \cdots\ f_{cg_k}\ \cdots\ f_{cg_{mp}}\right\}^T f_{cg_k} = \left\{f_{cg_{k\xi}}\ \ f_{cg_{k\eta}}\ \ f_{cg_{k\varsigma}}\right\}^T.$$

The relationship between f_{cg_k} and f_{cl_k} is

$$f_{cg_k} = rr_k^T f_{cl_k}, \quad (2)$$

where rr_k^T is the coordinate transfer matrix between the global and local coordinate systems.

Once the internal constraint forces and rigid movements are known, the relative displacement $\{\delta\}_i$, relative velocity $\{\dot{\delta}\}_i$, and relative acceleration $\{\ddot{\delta}\}_i$ ($i = 1,2,...n$) to centroid point in each block can be solved with general finite element method. That is,

$$\begin{aligned} &K_i\delta_i + D_i\dot{\delta}_i + M_i\ddot{\delta}_i = F_i + R_i - F_i^g \\ &F_i^g = K_i\delta_i^g + D_i\dot{\delta}_i^g + M_i\ddot{\delta}_i^g, \end{aligned} \quad (3)$$

where K_i is elasticity matrix, D_i is damping matrix, M_i is mass matrix, F_i is external load vector applied on the ith block, and R_i is load vector transferred from the internal force in discontinuous interface, F_i and R_i can be expressed as

$$\begin{aligned} F_i &= \{f_1, f_2, \cdots f_{np}\}_i^T \\ R_i &= \{r_1, r_2, \cdots r_{np}\}_i^T, \end{aligned}$$