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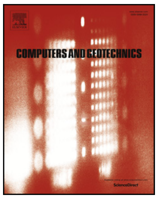

Research Paper

Random bearing capacity evaluation of shallow foundations for asymmetrical failure mechanisms with spatial averaging and inclusion of soil self-weight

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ABSTRACT

In this study, multi-block failure mechanisms in conjunction with Vanmarcke's spatial averaging approach are used to evaluate random bearing capacity. This new approach examines the impact of the asymmetrical mechanism in spatially variable soil and the effect of anisotropy. The soil strength parameters were modelled by random fields that were discretized by spatial averaging along slip surfaces. Reliability indices were evaluated for symmetrical and asymmetrical cases. The results show that for higher values of horizontal fluctuation scales, the difference between symmetrical and asymmetrical approaches becomes negligible; however, for smaller values, it can be significant.

1. Introduction

Spatial variability in material properties is a crucial factor in geotechnical engineering, which distinguishes it from other areas of civil engineering. Variability in soil parameters has a significant impact on the level of construction safety. The physical and mechanical parameters of soils vary randomly even within homogenous soil deposit layers. The reason for this lies in natural sedimentation and consolidation processes. In response to the spatial variability in soils, probabilistic methods can be used for reliability measures in geotechnics. Thus, recently, researchers have turned to probabilistic methods [1–5]. As a result, methods have been developed to deal with soil spatial variability, e.g., the random finite element method (RFEM) [4,6–8], random field limit analysis (RFLA) [9], and random adaptive finite element method (RAFELA) [10]. The kinematic method of limit analysis is also a powerful tool, and can be applied in conjunction with the probabilistic approach to evaluate bearing capacity [9–13]. In an earlier paper by the authors of this study [14], this approach was combined with Vanmarcke's spatial averaging [2,15,16]. This approach was motivated by the need for a more detailed description of soil properties based on random field theory. In the present study, the authors extend and improve the approach to analyse the multi-block asymmetrical mechanisms of possible failure considering the self-weight of the soil. According to limit analysis theory [17], the Prandtl mechanism [18] is optimal for weightless soil (the upper and lower bounds are equal). If soil weight is included, this approach is no longer valid. The resulting bearing capacity value is greater than the solutions obtained by, for example, the method of characteristics or by the Sokolovskii approach [19]. The influence of soil weight on bearing capacity in limit analysis was discussed by Michalowski [20]. An alternative approach was given by the creators of finite element limit analysis [21–24]. When the mass of the soil is considered in a multi-block failure mechanism, there is no direct solution to establish the geometry of failure; an optimization procedure is mandatory to find the minimum value of the upper bound load. The optimal geometry also has a significant impact on the averaging level, which generally depends on the slip line length (greater variance reduction [14]). As previously noted [14], an averaging procedure should be adopted for the volume of soil involved in the failure mechanism. Therefore, taking into account the optimized kinematically admissible failure mechanisms resulting from the kinematic method of limit analysis seems to be appropriate and rational. To make the optimization effective and precise, the authors choose the simulated annealing method [25,26], which has been successfully applied to geotechnical problems [27,28] and micromechanics [29,30].

Based on the behaviour of natural soil properties [31–34], anisotropy in the soil strength parameters is considered in this paper. According to experience, a large value for the horizontal scale of fluctuation is utilized, which means that the soil properties are more strongly correlated in the horizontal direction than in the vertical

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Nomenclature

N_c,N_q,N_γ	bearing capacity factors
c	cohesion
φ	angle of internal friction
c_i,φ_i	c and φ for specific slip line i in failure mechanism
$\bar{\varphi}_i,\bar{c}_i$	c and φ for specific slip line i in failure mechanism after averaging procedure
γ	unit weight of soil
q	overburden pressure
l_i,β_i	lengths and angles in failure geometry mechanism (i depends on number of blocks)
l_{0i},β_{0i}	initial geometry parameters (in simulated annealing)
l_{ci},β_{ci}	current geometry parameters (in simulated annealing)
g_i	gravitational forces
p	bearing capacity
p_c	current value of bearing capacity (in simulated annealing)
p_{new}	new value of bearing capacity (in simulated annealing)
p_{exp}	value of bearing capacity obtained for expected values of random parameters
p_{fit}	bearing capacity (random variable) of fitted log-normal distribution
v_i	velocity discontinuities on specific slip line ($v_{i\vert}$ – vertical component of v_i)
P_a	acceptance probability in simulated annealing
P{…}	probability of a certain event
$\alpha,T_{cur}, z,T_{min}$	simulation process controlling parameters (in simulated annealing)
U[0,1]	uniform distribution on the interval [0,1]
X_V	random field after averaging X in the domain V
σ_X^2	point variance of the property field X
σ_V^2	reduced variance of the property X in the domain V
$R()$	covariance function
θ_v,θ_h	vertical and horizontal fluctuation scale, respectively
$Var(),V()$	variance
$Cov(),C()$	covariance
x_A,z_A	Cartesian coordinates of point A
N	number of Monte Carlo algorithm realizations
b	width of foundation
$[C_X]$	covariance matrix
cdf	cumulative distribution function
pdf	probability density function
F	global safety factor
P_f	probability of failure
β	reliability index
Φ^{-1}	inverse function to cumulative distribution function of the standard normal distribution
RFEM	random finite element method
RFLA	random field limit analysis
RAFELA	random adaptive finite element method

direction.

The main objective of this paper is to examine the influence of the assumption of failure mechanism symmetry on reliability indices by comparison with an asymmetrical case, assuming that the self-weight of the soil is included. Asymmetry in the failure mechanism can appear due to spatial variation in soil properties and can be considered a typically random phenomenon. An asymmetrical mechanism has not been considered in earlier papers. A new probabilistic procedure is presented based on multi-block symmetrical and non-symmetrical mechanisms, which finally leads to the evaluation of the failure probability. This work takes into account the random nature of the slip line positions caused by random variations in the angle of internal friction and cohesion, and reliability evaluation is conducted for a variety of fluctuation scale values.

2. Geometry of failure surfaces and bearing capacity evaluation

2.1. Bearing capacity formula

This paper addresses a multi-block failure mechanism composed of rigid blocks that are separated by straight slip lines [35,20]. The limiting value of the bearing capacity formula originates from the kinematical theorem of limit analysis; which states that the rate of work by the external forces is less than (or equal) to the rate of energy dissipation in any kinematically admissible mechanism [20,35,36]. The bearing capacity formula commonly presented is a sum of three terms:

$$p = cN_c + qN_q + \frac{1}{2}\gamma bN_\gamma \tag{1}$$

where c is the cohesion, q is the overburden pressure, γ is the unit weight of soil and b is the width of the foundation. Thus, factors N_c, N_q and N_γ are associated with cohesion, overburden and soil self-weight, respectively. As long as $\gamma = 0$ (weightless soil), the first two factors are functions only of the angle of internal friction; however, when the weight of soil $\gamma \neq 0$, the values of N_c, N_q and N_γ are dependent on c, q, γ and b [20]. When the weight of the soil is included, the Prandtl solution is not exact according to the limit analysis theory; the value of N_γ obtained for the Prandtl mechanism is more conservative than the one obtained from the multi-block mechanism. However, the multi-block failure mechanism with the weightless soil assumption reaches the Prandtl solution with an increasing number of blocks. Detailed discussion on this subject can be found in [20]. In this paper, the multi-block

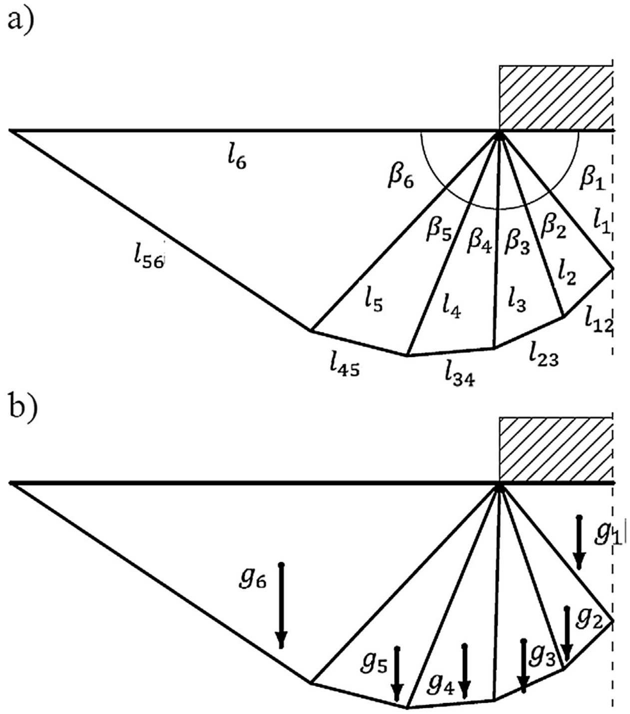

Fig. 1. Example of the geometry of a 6-block symmetrical failure mechanism (a) and gravitational forces (b).