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Laminar natural convection of power-law fluids in a square enclosure with differentially heated side walls subjected to constant temperatures

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ABSTRACT

Two-dimensional steady-state simulations of laminar natural convection in square enclosures with differentially heated sidewalls subjected to constant wall temperatures have been carried out where the enclosures are considered to be completely filled with non-Newtonian fluids obeying the power-law model. The effects of power-law index n in the range $0.6 \leqslant n \leqslant 1.8$ on heat and momentum transport are investigated for nominal values of Rayleigh number (Ra) in the range 10^3–10^6 and a Prandtl number (Pr) range of 10–10^5. It is found that the mean Nusselt number $\overline{Nu}$ increases with increasing values of Rayleigh number for both Newtonian and power-law fluids. However, $\overline{Nu}$ values obtained for power-law fluids with $n < 1$ ($n > 1$) are greater (smaller) than that obtained in the case of Newtonian fluids with the same nominal value of Rayleigh number Ra due to strengthening (weakening) of convective transport. With increasing shear-thickening (i.e. $n > 1$) the mean Nusselt number $\overline{Nu}$ settles to unity ($\overline{Nu} = 1.0$) as heat transfer takes place principally due to thermal conduction. The effects of Prandtl number have also been investigated in detail and physical explanations are provided for the observed behaviour. New correlations are proposed for the mean Nusselt number $\overline{Nu}$ for both Newtonian and power-law fluids which are shown to satisfactorily capture the correct qualitative and quantitative behaviour of $\overline{Nu}$ in response to changes in Ra, Pr and n.

1. Introduction

Natural convection in rectangular enclosures with differentially heated vertical sidewalls and adiabatic horizontal walls is one of the most extensively studied configurations for Newtonian flows [1–3]. The extensive review of Ostrach [4] neatly captures the available data up to that date. In addition to the obvious fundamental interest, this configuration has engineering relevance in solar collectors, food preservation, compact heat exchangers and electronic cooling systems. In comparison to the vast body of literature regarding the natural convection of Newtonian fluids, a comparatively limited effort has been directed towards understanding of natural convection of non-Newtonian fluids in rectangular enclosures. The Rayleigh–Bénard configuration [5], which classically involves a rectangular enclosure with adiabatic vertical walls and differentially heated horizontal walls with the bottom wall at higher temperature, has been investigated for a range of different non-Newtonian models including inelastic Generalised Newtonian Fluids (GNF) [6–9], fluids with a yield stress [10–12] and viscoelastic fluids [13].

Kim et al. [14] studied transient natural convection of non-Newtonian power-law fluids (power-law index $n \leqslant 1$) in a square enclosure with differentially heated vertical side walls subjected to constant wall temperatures. They studied a range of nominal Rayleigh numbers from $Ra_K = 10^5$–10^7 and Prandtl numbers from $Pr_K = 10^2$–10^4 and demonstrated that the mean Nusselt number $\overline{Nu}$ increases with decreasing power-law index n for a given set of values of Ra_K and Pr_K.[1] This result is consistent with the numerical findings of Ohta et al. [8] where the Sutterby model was used for analysing transient natural convection of shear-thinning fluids in the Rayleigh–Bénard configuration. The augmentation of the strength of natural convection in rectangular enclosures for shear-thinning fluids was also confirmed by both experimental and numerical studies on micro-emulsion slurries by Inaba et al. [9] in the Rayleigh–Bénard configuration. Lamsaadi et al. [15,16] have studied the effects of the powerlaw index on natural convection in the high Prandtl number limit for both tall [15] and shallow enclosures [16] where the sidewall boundary conditions are subjected to constant heat fluxes (rather than isothermal as in the cases discussed above). Lamsaadi

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[1] The definitions of Ra_k and Pr_k are provided later in Section 2.

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Nomenclature

c_p	specific heat at constant pressure (J/kg K)
e	relative error (–)
e_{ij}	rate of strain tensor (s^{-1})
F_s	factor of safety (–)
g	gravitational acceleration (m/s^2)
Gr	Grashof number (–)
h	heat transfer coefficient (W/m^2 K)
K	consistency ($N s^n/m^2$)
k	thermal conductivity (W/m K)
L	length and height of the enclosure (m)
n	power-law index (–)
Pr	Prandtl number
q	heat flux (W/m^2)
r	ratio between the coarse to fine grid spacings (–)
r_e	grid expansion ratio (–)
Ra	Rayleigh number (–)
T	temperature (K)
t	time (s)
u_i	ith velocity component (m/s)
U,V	dimensionless horizontal ($U = u_1L/\alpha$) and vertical velocity ($V = u_2L/\alpha$) (–)
ϑ	characteristic velocity (m/s)
x_i	coordinate in ith direction (m)
α	thermal diffusivity (m^2/s)
β	coefficient of thermal expansion (1/K)
δ,δ_{th}	velocity and thermal boundary-layer thickness (m)
θ	dimensionless temperature ($\theta = (T - T_C)/(T_H - T_C)$) (–)
μ	dynamic viscosity ($N s/m^2$)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
τ_{ij} (τ)	stress tensor (stress) (Pa)
ϕ	general primitive variable (–)
ψ	dimensionless stream function (–)
Subscripts	
a	apparent
C	cold wall
$char$	characteristic value
eff	effective value
ext	extrapolated value
H	hot wall
K	based on definitions given in [14]
max	maximum value
nom	nominal value
ref	reference value
$wall$	wall value
Special characters	
ΔT	difference between hot and cold wall temperature (= (T_H-T_C)) (K)
$\Delta_{\min,cell}$	minimum cell distance (m)

et al. [15,16] show that the convective heat transfer rate becomes dependent only on nominal Rayleigh number Ra and the power-law index n for large values of aspect ratio and the nominal Prandtl number Pr.[1] Barth and Carey [17] utilised GNF models which incorporate limiting viscosities at low and high shear rates to study a three-dimensional version of the problem (the adiabatic boundary conditions are replaced by a linear variation in temperature to match the experimental conditions of [18]). Recently Vola et al. [19] and the present authors [20,21] numerically studied steady two-dimensional natural convection of yield stress fluids obeying the Bingham model in rectangular enclosures with differentially heated vertical side walls and proposed correlations for the mean Nusselt number $\overline{Nu}$.

In the present study steady natural convection of Ostwald–De Waele (i.e. power-law) fluids in a square enclosure with differentially heated side walls subjected to constant wall temperatures has been studied numerically. A parametric study has been conducted with the power-law index n ranging from 0.6 to 1.8 for a range of nominal values of Rayleigh and Prandtl numbers (definitions are provided in Section 2) given by $Ra = 10^3–10^6$ and $Pr = 10–10^5$. The simulation data in turn has been used to develop a correlation for the mean Nusselt number $\overline{Nu}$ based on a detailed scaling analysis for the broad range of n, Ra and Pr considered in this study. In this respect the main objectives of the present paper are as follows:

(1) To demonstrate the effects of n, Ra and Pr on the mean Nusselt number $\overline{Nu}$ in the case of natural convection of power-law fluids in a square enclosure with differentially heated vertical side walls subjected to constant wall temperatures.
(2) To elucidate the above effects with the aid of a detailed scaling analysis.
(3) To develop a correlation for the mean Nusselt number for natural convection of power-law fluids in a square differentially heated vertical side walls subjected to constant wall temperatures.

The rest of the paper will be organised as follows. The necessary mathematical background and numerical details will be presented in the next section, which will be followed by the scaling analysis. Following this analysis, the results will be presented and subsequently discussed. The main findings will be summarised and conclusions will be drawn in the final section of this paper.

2. Mathematical background and numerical implementation

2.1. Non-dimensional numbers

For the Ostwald–De Waele (i.e. power law) model the viscous stress tensor τ_{ij} is given by:

$$\tau_{ij} = \mu_a e_{ij} = K(e_{kl}e_{kl}/2)^{(n-1)/2} e_{ij}, \quad (1)$$

where $e_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ is the rate of strain tensor, K is the consistency, n is the power-law index and μ_a is the apparent viscosity which is given by:

$$\mu_a = K(e_{kl}e_{kl}/2)^{(n-1)/2}. \quad (2)$$

For $n < 1$ ($n > 1$) the apparent viscosity decreases (increases) with increasing shear rate and thus the fluids with $n < 1$ ($n > 1$) are referred to as shear-thinning (shear-thickening) fluids. In the present study, natural convection of power-law fluids in a square enclosure (of dimension L) with differentially heated constant temperature side walls filled with power-law fluids is compared with the heat transfer rate obtained for different values of n with the same nominal values of Rayleigh number and Prandtl number. The nominal Rayleigh number Ra_{nom} represents the ratio of the strengths of thermal transport due to the buoyancy force to that due to thermal diffusion, which is defined here as:

$$Ra_{nom} = \frac{\rho^2 c_p g \beta \Delta T L^3}{\mu_{nom} k} = Gr_{nom} Pr_{nom}, \quad (3)$$