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Research Paper

Stochastic analysis of three-dimensional hydraulic conductivity upscaling in a heterogeneous tropical soil

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ABSTRACT

Hydraulic conductivity (K) heterogeneity is seldom considered in geotechnical practice for the impossibility of sampling the entire area of interest and for the difficulty of accounting for scale effects. Stochastic three-dimensional K upscaling can tackle these two problems, and a workflow is described with an application in a tropical soil. The application shows that K heterogeneity can be incorporated in the daily practice of the geotechnical modeler while discussing the aspects to consider when performing the upscaling so that the upscaled models reproduce the average fluxes at the fine scale.

1. Introduction

Hydraulic conductivity (K) is one of the most important parameters in many geotechnical studies such as when analyzing slope stability; the dewatering of an underground excavation the design of an earth dam; or the analysis of seepage, flow, and contaminant transport in liners and embankments. Most of these problems are approached using numerical simulations, where K is a key input parameter, the heterogeneity of which plays an important role even in apparently homogeneous soils [1–6]. However, the use of heterogeneous K fields in numerical modeling in geotechnical engineering is an exception rather than a rule [7–9] because, in general, deterministic approaches that consider K as a constant value for an entire soil layer are employed [1,10–12]. The impossibility of sampling the entire area of interest together with the difficulty of accounting for scale effects [6,13–17] are the two main reasons why heterogeneity is not accounted for in practice. This paper tries to address these two problems and describes how to cope with them.

To face the problem of having scarce information for a complete description of the heterogeneity of K, we use geostatistical techniques such as stochastic simulation or kriging estimation, which permit a coherent assignment of values at locations where measurements were not taken, based on the values observed at measurement locations [18–22]. Whether to employ simulation or estimation will depend on the use to be given to the generated maps.

The coherent assignment of values mentioned above does not remove the uncertainty associated with having limited information about the spatial variability of K in the area of interest; a model of uncertainty is needed, which is built on the framework of stochastic random fields [23]. Hydraulic conductivity will be modeled as a random field, that is, as a set of spatially correlated random variables. At each location in space, K is modeled as a random variable with a probability density function (pdf) rather than a unique value; the pdf represents the likelihood that K takes a specific value at that location [20]. It is important to emphasize that K is not a result of a random process, but the concept of random field is a convenient modeling approach to formalize the problems of estimation and simulation. The random field is fully described by a multivariate pdf, which, in turn, is described by a series of parameters such as the mean, variance, autocorrelation, or variogram. In the past years, the number of researchers in geotechnical engineering, who deal with K heterogeneity in a stochastic way, has increased, but deterministic analysis still prevails [9,24–28].

To face the problem of scale effects, recall that in geotechnical practice, K is measured at the field or laboratory on a support of around a few centimeters [29,30]. Then, those K values are used to feed the K values of a numerical model, where the discretization support is generally orders of magnitude larger than the measurement support [31]. The change of support (from the measurement scale or fine scale to the numerical scale or coarse scale) implies a change in the properties of the random field. The use of some upscaling technique that transfers the information obtained at the fine scale into the coarse scale to be used by

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the numerical code is necessary to deal with the discrepancy between laboratory and numerical scales [19,32,33]. In other words, the upscaling process seeks a block conductivity (K_V) that preserves the total flow crossing the block observed in the block of heterogeneous cell conductivities (K_f) for the same hydraulic head gradient. During the transfer between scales, there is a loss of information because the small-scale heterogeneity is not preserved; however, the fluxes occurring at the coarse scale should be the same as those obtained, had the domain being modeled as fully heterogeneous at the small scale. To determine the block conductivity is not a simple task. It should be remembered that the block conductivity, as defined above, is not the arithmetic average of the cell values within the block, which is a common geotechnical practice to upscale K when only a few measurements are available [34].

Many authors had worked to improve the upscaling methods, which range from simple averaging to the Laplacian-with-skin method with uniform and nonuniform coarsening. They have achieved very good results, showing some advantages, limitations, and the evolution of the K upscaling techniques in a variety of problems [18,32–48]. In addition, some relevant works associated with geotechnical engineering showed that coupled approaches should be used in the upscaling of soil properties to properly model some of the behaviors of heterogeneous soils, e.g., consolidation [9,49]. There are also complete reviews on saturated K upscaling methods [31,50,51], and the reader is encouraged to read these papers. The nomenclature used hereafter to refer to the different upscaling approaches is taken from the Ref. [31]. Some conclusions found in the literature are that the K upscaling is site specific and depends on the boundaries conditions, block size and shape, statistical isotropy, block size relative to the correlation length, dimensionality of the problem, and complexity of the studied environment. Once the problem of upscaling is resolved, one should not forget that cell values (from which the block conductivities are computed) are never exhaustively known; therefore, it is necessary to quantify the uncertainty associated with the upscaled values using a stochastic approach [48].

In this paper, we would like to focus on two upscaling methods, a simple averaging method, specifically the empirical power average [52] or p-norm, and the Laplacian-with-skin method [48]. The former method has advantages such as usefulness, simplicity, and widespread use [38,39,43,52–54], while the latter method has advantages such as robustness and very good reproduction of the fine scale flows at the coarse scale [18,19,32,48].

It is important to stress that almost all the background information provided here was developed in petroleum engineering and hydrogeology. Very few studies associated with K upscaling have been found in the geotechnical engineering literature [9,47,49,55]; to the best of our knowledge, the more sophisticated Laplacian-based upscaling methods have not yet been applied in geotechnical engineering. Tropical soils have a very specific behavior and are a source of many geotechnical problems; this paper presents, for the first time, an application of K upscaling to this type of soil.

The power-average method was used to upscale K for a unique block size for a 3D anisotropic real aquifer [32] and for a bi-dimensional hypothetical aquifer [38]. Power average was also used to determine K_V for a range of block shapes for synthetic cases [39]. In the last two works, the exponent of the power average was determined based on numerical experiments. The simple-Laplacian technique was used in a bi-dimensional conceptual model based on data from a real site in the context of nuclear waste disposal [20]. K upscaling by the Laplacian-with-skin method was applied in a realization of a three-dimensional synthetic K field [18]. This technique was also used to determine K_V for three block sizes in a bidimensional numerical example, after solving the flow equation by a finite-difference numerical model with the approximation of the interblock conductivity [18].

To summarize, this paper has three objectives: (i) an analysis of stochastic 3D hydraulic conductivity upscaling using the Laplacian-with-skin method [48] for a variety of block sizes using real K measurements obtained in a tropical soil in Brazil; (ii) to demonstrate the errors that can be introduced by using a deterministic upscaling using harmonic, arithmetic, and geometric averages of the measured K without accounting for the spatial correlation; and (iii) to show how and when the p-norm averaging can be used (for the tropical soil studied) as an alternative to the more complex and time-consuming Laplacian-with-skin method, with the aim of providing a practical and fast solution for the daily practice of the geotechnical modeler. As a by-product of this third objective, the dependence of the exponent of the p-norm as a function of the block size is analyzed.

2. Hydraulic conductivity upscaling methods

The main objective of upscaling is to obtain a block K_V value that reproduces the groundwater flow at the coarse scale as if it had been computed at the fine scale. The aim is to replace a finely discretized heterogeneous spatial distribution of conductivities K_f, with a set of block values K_V, so that the flow response of the set of coarse block values matches the response at the fine scale.

Upscaling methods can be classified as local and nonlocal [31]. Simple averaging techniques are local methods and assume that K_V depends only on the K_f values within the block [35,37,56]. For a perfectly layered soil, it can be shown that K_V is equal to the harmonic mean (K_h) of the cell conductivities inside the block when the flow is perpendicular to the layers, and to the arithmetic mean (K_a) when the flow is parallel to them [56]. It can also be shown that for 2D flow in an isotropically heterogeneous field with lognormally distributed conductivities, K_V is equal to the geometric mean (K_g) of the cell conductivities [35,36]. For 3D-flow, there is no closed form for the best average process because it will depend on the statistical isotropy and the spatial correlation structure [31] of the cell conductivities.

It is well established that K_V must be between the arithmetic mean and the harmonic mean [37]. The p-norm average was proposed as a flexible easy-to-compute alternative because it can provide a value for K_V between those two limits as a function of the exponent p [52]:

$$K_{V,p} = \left(\frac{1}{V}\int_V K_f^p(\mathbf{u})d\mathbf{u}\right)^{\frac{1}{p}} \tag{1}$$

where V indicates the volume of the block; $K_{V,p}$ is the block conductivity determined using the p norm, and K_f represents the cell conductivities within the block. The power p is allowed to vary between -1 and $+1$. When p is equal to -1, $K_{V,p}$ equals K_h; when p is equal to 0, $K_{V,p}$ equals K_g; and when p is equal to $+1$, $K_{V,p}$ equals K_a. The challenge of p-norm upscaling is to determine the exponent p that will result in a $K_{V,p}$ that reproduces the flows observed at the fine scale. The p-norm is a very practical method that can provide very good results in some situations [1,38,39]. In cases where the degree of heterogeneity is mild, simple averaging methods compete favorably with more sophisticated methods [53]. However, the p-norm average cannot be used without resorting to some prior numerical modeling to find the best p exponent [39].

K_V depends not only on the cell values of flux and hydraulic head but also on the boundary conditions around the block; the fact that the same layered block will have different upscaled block values depending on whether the flow is parallel or orthogonal to it proves it. K_V is said to be nonlocal [31], i.e., it depends not only on the cell values within the block but also on external factors. The simple-Laplacian is a nonlocal approach [31,48] that was developed to deal with the need to determine K_V considering the boundary conditions that are acting on the block boundaries. The introduction of this method represented a big improvement of the upscaling techniques when compared to local methods. Nevertheless, in this approach, the principal components of K_V are assumed to be parallel to the block sides and the boundary conditions used to solve the flow at the fine-scale do not necessarily coincide with the real boundary conditions that the block may have