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Research Paper

A generalized contact potential and its application in discontinuous deformation analysis

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ABSTRACT

The distributed contact force that is determined from the contact potential can be used to deal with the contact between discrete bodies. The current study defines the generalized contact potential (GCP). The GCP is independent of the shape and size of body and possesses the thorough geometric locality. If and only if the contact regions are exactly the same, the contact potentials are identical. By combining discontinuous deformation analysis (DDA) and GCP a new DDA, named GCP-DDA, is produced, in which the generalized-α method is adopted to discretize the time domain. The GCP-DDA can make the global controlling equation and the open-close iteration (OCI), which may cause the reduction of time step size and the rebuilding and solving of global controlling equation, become unnecessary. The intractable issues related to convex-convex contact in the original DDA can be accordingly bypassed.

1. Introduction

In the field of science and engineering, generally speaking, there are two different kinds of methods: the continuum-based method and the discontinuum-based method. In the former approaches, the problem domain is usually subdivided into a finite number of sub-domains (elements). The continuity condition must be satisfied at those interfaces with adjacent sub-domains. To capture discontinuities in the framework of continuous-based method, some new approaches and techniques were presented [1–7], to name just a few recent developments. On the other hand, among the latter approaches to simulate directly the behaviour of a discrete block system, in which various modes of motion such as sliding, rotating, opening, and locking maybe entangled with each other, the distinct element method (DEM) [8] should be firstly mentioned. In the DEM, the basis of the kinematics of blocks is the Newton's second law and the DEM employs the explicit time integration scheme, which is based on the finite difference approximation. For the problems containing a large number of separate particles or blocks, it has been recognized that the DEM enjoys a higher computational efficiency. It is generally believed that no need to solve the global simultaneous equilibrium equations is the main reason [9]. The development of this method can be reflected to some extent in the process of commercialization of the DEM's code [10–12]. The DEM has a wide range of applications but despite that the degeneration of accuracy of results given by DEM has been discussed in some particular cases [13].

Another discontinuum-based method is the discontinuous deformation analysis (DDA) [14,15]. As a bridge between finite element method (FEM) and limit equilibrium method (LEM) [16], DDA is characterized by the several features as follows: the principle of minimum potential energy, based on which the DDA satisfies strict equilibrium at each time step [16]; the implicit time integration scheme, which can more ensure the stability of numerical computation when a larger time step size is used; the simplex integration method, which is a kind of analytic integral method irrespective of the shape of block and can more ensure the accuracy of results; the open-close iteration (OCI) contact algorithm, which imposes the contact conditions.

Over a period of 30 years, a lot of efforts have been made to DDA. Some convergence criteria [17] and the strategy of strengthening the movement trend [18] were proposed to speed up the convergence of the OCI. The post-adjustment [19], the trigonometric [20], the Taylor series [21], the post-contact adjustment [22] and the displacement-strain modification [23] were suggested to overcome the volume expansion of block. The higher-order DDA [24], a nodal-based DDA [25,26], and the FEM-DDA [27] were used to enhance the ability to cope with the deformation of structure. The one temporary spring method [28] and the angle-based method [18] were used to tackle the indeterminacy of vertex-vertex contact. In order to improve the accuracy of contact force, the Lagrange multiplier method [29], the

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augmented Lagrange multiplier method [30], the variational inequality method [31,32] and the complementarity method [33–35] were also considered. Recently, the strain-rotation decomposition theorem was introduced into the DDA [36,37].

In comparison with the DEM, the original DDA suffers from a high computational cost due to the implicit time integration scheme (Newmark β method [38] when $\beta = 1/2, \gamma = 1$) and the OCI. The global controlling equation must be assembled and then solved in the standard DDA. Along with the increase in the number of blocks, the size of global governing equation is enlarged. Thus, the computational consumption increases further. On the other hand, to achieve no-penetration and no-tension at each contact point the OCI is always need to carry out repeatedly at each time step. This further results in higher computational cost. Besides, the reduction of the time step size during the OCI may offset the advantage of a large time step size, which is allowed by the implicit time integration scheme.

Because any block does not share the basic unknown variables with other blocks, which is very different from the degrees of freedom (DOFs) of nodes in FEM, if there are not contacts in the block system, the assembly of the global governing equation can be regard as the process of "seating by number" according to the serial number of blocks. Whenever a contact occurs, in the original DDA the contact force is assumed to be concentrated, which makes the process of "seating by number" become complex. If a distributed contact force is employed, the conciseness of the process of "seating by number" can still be retained. In this case, more importantly, from the viewpoint of solving equation system, the global governing equation does not be needed due to the fact that the participants of the assemblage are independent of each other. In this study, the definition of generalized contact potential (GCP) is proposed. The GCP is independent of the shape of blocks, possesses thorough locality, can lead to a distributed contact force. The GCP can be employed by the DEM or FEM/DEM and can be seen as the generalization of the existing contact potential [39], which is specially aimed at triangular block or element and has been used into the numerical manifold method [40]. It is worth mentioning that for triangular element a modified definition and calibration proposed by [41]. Subsequently, applying the GCP into the DDA, a GCP-DDA is established, in which the implicit generalized-α method [42] is adopted, and the intractable issue associated with convex-convex contact can be easily avoided.

2. Generalized contact potential

The distributed contact force is in general evaluated from the contact potential, which depends on the shape and size of overlap between the blocks in contact. In this section, the generalized contact potential will be described in detail. Based on the fact that for a pair of contact the overlap region only occupies a small portion zone, which is belonged to the two blocks in contact at the same time. Complying with the original DDA [14], for the two-dimensional case, all types of contact can be classified into four categories: the convex-convex contact, the convex-concave contact, the convex-edge contact, and the edge-edge contact, as shown in Fig. 1.

According to [39], for two blocks in contact one of which is denoted as the contactor and the other as the target, when in contact the contactor and target overlap each other over area S, bounded by boundary Γ (Fig. 2).

Consider infinitesimal overlap $\mathrm{d}A$ defined by overlap points $\mathrm{P_c}$ belonging to the contactor and $\mathrm{P_t}$ belonging to the target, the infinitesimal contact force $\mathrm{d}\mathbf{f}$ is given by

$$\mathrm{d}\mathbf{f} = [\mathrm{grad}\varphi_c(\mathrm{P_c}) - \mathrm{grad}\varphi_t(\mathrm{P_t})]\mathrm{d}A \tag{1}$$

where φ_c and φ_t are contact potentials for the contactor block and target block, respectively. Further, Eq. (1) can be expressed as

$$\mathrm{d}\mathbf{f} = \mathrm{d}\mathbf{f}_c - \mathrm{d}\mathbf{f}_t \tag{2}$$

where

$$\begin{aligned} \mathrm{d}\mathbf{f}_c &= -\mathrm{grad}\varphi_t(\mathrm{P_t})]\mathrm{d}A_c, \quad \mathrm{d}A_c = \mathrm{d}A \\ \mathrm{d}\mathbf{f}_t &= -\mathrm{grad}\varphi_c(\mathrm{P_c})]\mathrm{d}A_t, \quad \mathrm{d}A_t = \mathrm{d}A \end{aligned} \tag{3}$$

The contact as described by Eq. (3) can be viewed firstly as the elemental area of the contactor penetrating the target and then the elemental area of the target penetrating the contactor [39]. For each of the two blocks in contact, the contact force is calculated as the gradient of the corresponding contact potential. The field of contact forces is therefore a conservative field for both the target penetrating the contactor and the contactor penetrating the target. A helpful conclusion has been proved by [39]: the contact potential on the boundary of both contactor and target blocks are constant, the contact force given by Eq. (1) preserves the energy balance regardless of the geometry or the shape of contactor and target blocks, size of the penalty term or size of penetration (overlap).

The total contact force can be obtained by calculating the integration of Eq. (1) over the overlap area S

$$\mathbf{f}_c = p_n \int_{s=\mathrm{B_t}\cap\mathrm{B_c}} [\mathrm{grad}\varphi_c(\mathrm{P_c}) - \mathrm{grad}\varphi_t(\mathrm{P_t})]\mathrm{d}A \tag{4}$$

which can also be written as an integral over the boundary Γ of the overlap area [39]

$$\mathbf{f}_c = p_n \int_{\mathrm{B_t}\cap\mathrm{B_c}} \mathbf{n}(\varphi_c - \varphi_t)\mathrm{d}\Gamma \tag{5}$$

where $\mathbf{n}$ is the outward unit normal to the boundary of the overlap area. p_n is the penalty term.

For any point P in a triangular element or block, as shown in Fig. 3, the contact potential based on the area is defined as follows [39]:

$$\varphi(\mathrm{P}) = \min\left\{\frac{3A_1}{A}, \frac{3A_2}{A}, \frac{3A_3}{A}\right\} \tag{6}$$

where $A_i (i = 1, 2, \text{and } 3)$ is the area of the sub-triangle constructed by point P and one of the three sides of the triangular block or element, and A is the area of the triangular block or element. It has been reported that the definition in Eq. (6) can be lead to a different contact potential even if the embedding amount between two triangles is the same [41]. To eliminate the undesired phenomenon, the contact potential was redefined by the relevant heights [41], reading

$$\varphi(\mathrm{P}) = \frac{1}{H}\min\{h_{p-12}, h_{p-23}, h_{p-31}\} \tag{7}$$

where H is a standard embedded amount to make the potential is dimensionless, and $h_{\text{p-12}}$, $h_{\text{p-23}}$, and $h_{\text{p-31}}$ are the distances between point P and edges 12, 23, and 31, respectively, as shown in Fig. 4.

The definition by Eq. (7) can ensure that the contact force will be the same as long as the relevant embedding amount is identical. However, it is still limited to the triangular block or element.

Now that we have known that the total contact force caused by the overlap region can be converted into the equivalent contact force resulted from the boundary of the same overlap region, then the problem can be reduced to how to determine the contact potential on the boundary. Next, we will give a new definition for calculating the contact potential on the boundary.

First, let us see the convex-convex contact, as shown in Fig. 5. Vertexes $\mathrm{V_c}$ and $\mathrm{V_t}$ belong to the contactor $\mathrm{B_c}$ and target $\mathrm{B_t}$, respectively. Points A and B are the intersection points, namely, polygon $\mathrm{AV_tBV_c}$ represents the overlap region. The linear distributed contact potential can be obtained as follows.

(1) For points A and B the contact potentials are set to

$$\varphi(\mathrm{A}) = \varphi(\mathrm{B}) = 0 \tag{8}$$

(2) For point $\mathrm{V_t}$ of the target $\mathrm{B_t}$, set the contact potential exerted by target $\mathrm{B_t}$ is