Computers and Geotechnics 99 (2018) 130–136

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Technical Communication

Influence of boundary constraints on stress heterogeneity modelling

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ARTICLE INFO

Keywords:
Boundary stiffness
Stress dispersion
Effective variance
Stress heterogeneity
Finite-discrete element method

ABSTRACT

By employing the effective variance of stress tensors as a scalar-valued measure of stress heterogeneity, we quantitatively analyse the influence of boundary constraint stiffness on numerically derived stress distribution in a fractured rock mass. The results reveal a decreasing trend in the effective variance of stress field with an increasing boundary constraint stiffness. This work demonstrates the efficacy of effective variance for stress heterogeneity quantification, and also indicates that the boundary constraint stiffness can affect stress modelling results. We suggest that quantitative evaluation of the effects of boundary constraints may be needed in geomechanical modelling of fractured rock masses.

1. Introduction

Crustal rocks, embedded with widespread natural fractures, are subjected to stresses, mainly due to the overburden and tectonic effects [1]. Thus, the *in situ* state of stress is an important parameter for a wide range of endeavours in rock mechanics [1–6]. Because of the inherent complexity of fractured rock masses in terms of varying rock properties and presence of discontinuities, the stress state often exhibits significant heterogeneity [4,7–10]. The *in situ* stress measurement results shown in Fig. 1 exemplify the dramatic variation in both the principal stress magnitude and orientation along two sides of a fault [8].

However, a thorough characterisation of stress heterogeneity in the field is very challenging, which requires sufficient and detailed *in situ* stress measurements [11]. Due to implementation difficulties and budget limits, it is often difficult to conduct a large number of stress measurements in real engineering projects. Numerical simulation provides an alternative and fast solution to this issue [11,12]. In the past few decades, many numerical models have been developed to solve different rock mechanics problems [13–15], while only a few efforts have been devoted to investigating the phenomena of stress heterogeneity [11,16–21]. In these previous geomechanical modelling studies, different types of numerical boundary constraints, e.g. stress boundary constraints [3,16,21–25], displacement boundary constraints and combined stress-displacement boundary constraints [17,18,26–28], have been assumed for simulating the geological confinement imposed by surrounding rocks onto the problem domain (e.g. Fig. 2). It is found that rare discussions were made regarding the influence of different boundary constraint types on simulation results, which needs to be examined in a quantitative manner.

In order to quantify the variability of stress tensor fields, Gao and Harrison [29,30] proposed a stress variability characterisation approach using "effective variance" as a scalar-valued measure of the overall stress heterogeneity. This metric for stress tensor data has the similar functionality to the variance and standard deviation of scalar data. This effective variance approach has proven its accuracy and robustness in quantifying stress heterogeneity in complex geological media [29–31].

In this paper, we use the two-dimensional (2D) finite-discrete element method (FEMDEM) [32,33] to simulate the stress distribution in a fractured rock mass subjected to different types of boundary constraints. We employ the effective variance method to quantify the influence of boundary constraint stiffness on the simulated stress results. We aim to draw attention from the community to the potentially important effects of boundary constraint on geomechanical modelling. In the rest of the paper, we first introduce the effective variance method in Section 2, followed by a brief description of the FEMDEM approach in Section 3. We then present the model setup and simulation results in Section 4. Finally, a few concluding remarks are presented.

2. Effective variance – scalar-valued stress dispersion quantification

As mentioned earlier, stress in rock masses often displays significant heterogeneity. It is important that such heterogeneity can be characterised in a quantitative manner [34–37]. Dispersion, which denotes how scatter or spread out a data group is with respect to its mean, is an

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https://doi.org/10.1016/j.compgeo.2018.03.003
Received 8 November 2017; Received in revised form 11 February 2018; Accepted 11 March 2018

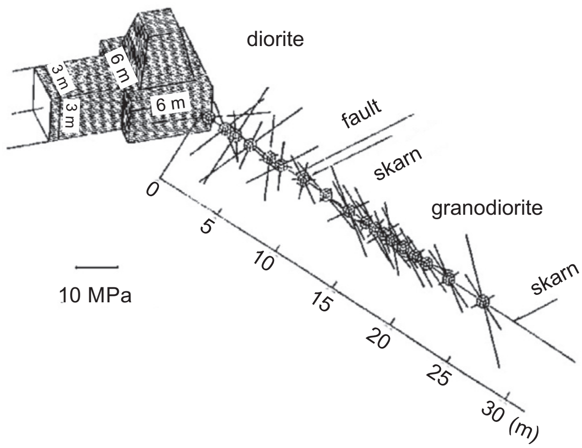


Fig. 1. Dramatic stress change observed near a fault. Note that the pairs of orthogonal intersecting lines represent the principal stress orientations and their length denote the principal stress magnitudes at different locations [8].

effective parameter for such characterisation. However, stress is tensor in nature formed by six distinct components. The conventional decoupled analysis of principal stress magnitude and orientation, which was usually adopted in the literature [38–41], may lead to biased assessment results [29,31,34,36,42,43].

To tackle this problem, considering that the variability of stress tensors can be adequately represented by the variability of its distinct tensor components in a multivariate manner [43], Gao and Harrison [29,30] proposed to employ the concept of "effective variance" for stress variability characterisation. The method of effective variance originated from the research field of multivariate statistics for group dispersion measure [44]. The effective variance of stress tensors can be calculated based on the covariance matrix of their distinct tensor components referred to a common Cartesian coordinate system. The detailed procedure is described as follows.

For a stress tensor

$$\mathbf{S} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ & \sigma_y & \tau_{yz} \\ \text{symmetric} & & \sigma_z \end{bmatrix}, \tag{1}$$

its distinct tensor components can be obtained as

$$\begin{aligned} \mathbf{s}_\mathrm{d} &= \mathrm{vech}(\mathbf{S}) = [\sigma_x \;\; \tau_{yx} \;\; \tau_{zx} \;\; \sigma_y \;\; \tau_{zy} \;\; \sigma_z]^T \\ &= [\sigma_x \;\; \tau_{xy} \;\; \tau_{xz} \;\; \sigma_y \;\; \tau_{yz} \;\; \sigma_z]^T \end{aligned}. \tag{2}$$

Here, the subscript "d" denotes "distinct", $[\cdot]^T$ represents the matrix transpose, and $\mathrm{vech}(\cdot)$ is the half-vectorisation function which stacks only the lower triangular (i.e. on and below the diagonal) columns of a tensor into column vector containing only its distinct components [45, p. 246]. For the stress vector $\mathbf{s}_\mathrm{d}$, its covariance matrix is

$$\mathbf{\Omega} = \mathrm{cov}(\mathbf{s}_\mathrm{d}) = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{s}_{\mathrm{d}_i} - \bar{\mathbf{s}}_\mathrm{d})\cdot(\mathbf{s}_{\mathrm{d}_i} - \bar{\mathbf{s}}_\mathrm{d})^T, \tag{3}$$

where $\bar{\mathbf{s}}_\mathrm{d}$ denotes the mean vector and can be calculated by

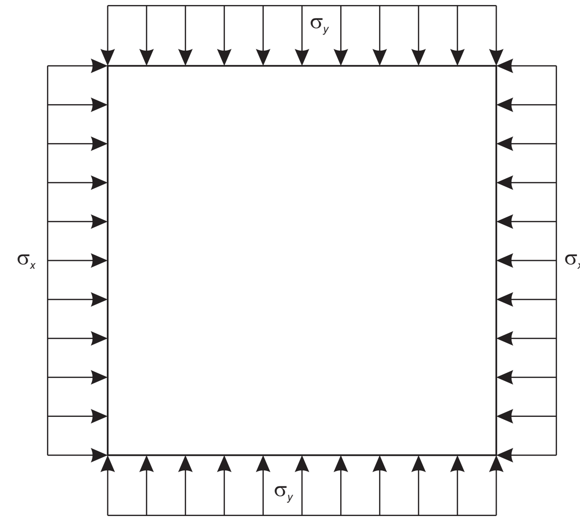
$$\bar{\mathbf{s}}_\mathrm{d} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{s}_{\mathrm{d}_i}. \tag{4}$$

Based on the covariance matrix $\mathbf{\Omega}$ given in Eq. (3), the effective variance is defined as

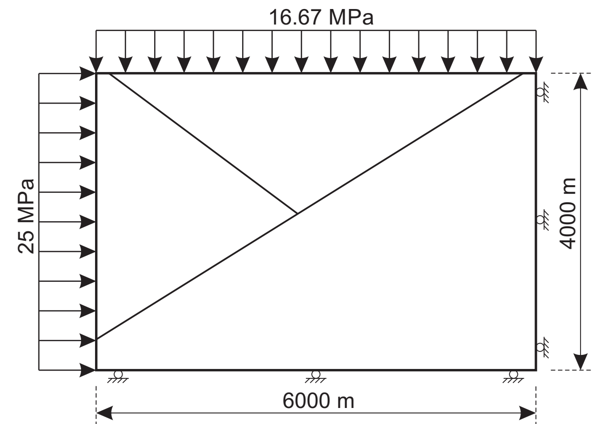
$$V_{\mathrm{e}|\mathrm{d}} = \sqrt[\frac{1}{2}p(p+1)]{|\mathbf{\Omega}|}, \tag{5}$$

where $|\cdot|$ denotes the matrix determinant and p ($p = 2$ or 3) is the dimension of the stress tensor.

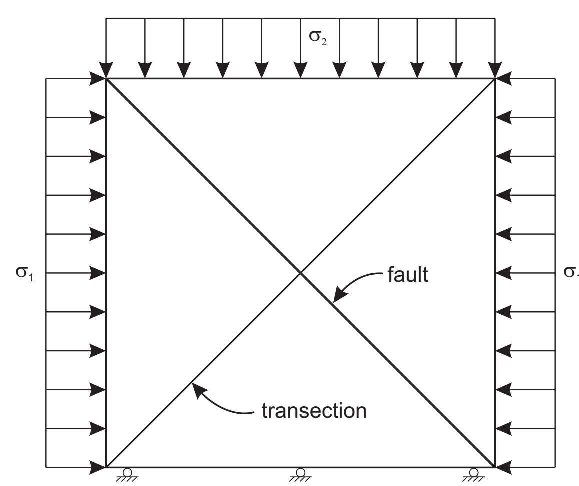
The effective variance has the same unit as the variance of the stress tensor components, i.e. square of the unit of stress. Similar to the variance and standard deviation of scalar data, the larger the effective variance, the more dispersed the stress tensor data would be.



(a) Boundary loading acting directly on the rock model [3, 16, 21-25]



(b) Direct loading and roller boundary on the rock model [17]



(c) Direct loading and roller boundary on the rock model [18, 26]

Fig. 2. Various boundary constraints have been used in geomechnical modelling in the literature.