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Research Paper

Sequential back analysis of spatial distribution of geomechanical properties around an unlined rock cavern

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ABSTRACT

This paper develops a sequential displacement data collection and back analysis approach for mapping spatially distributed Young's modulus (E) in a rock mass during the excavation of an unlined rock cavern (URC). Results show that this approach provides an unbiased estimate of the E field and its uncertainty. It also reveals a more detailed E distribution than kriging approach, which is based on samples of E values from boreholes before excavations. Further, predicted shear strain distribution and displacement of cavern periphery based on the estimates from this approach are more accurate than those based on the kriging estimate.

1. Introduction

Geomechanical properties, such as Young's modulus, E, the cohesion and the internal friction angle, play important roles in the stability analysis of unlined rock caverns (URCs). Many shallow URCs are constructed in soils or highly weathered and fractured rocks, where heterogeneity is significant and could have notable influences on the stability of URCs [4,41,44,43,9,45,20]. While the multi-scale variability of these properties at a field site is the rule rather than the exception, the variability is difficult to be characterized fully. In spite of this difficulty, many characterization approaches have been developed. One of the approaches is the inverse modeling or back analysis, which utilizes monitored displacements and/or stresses of the geological formations during a construction stage to estimate the parameters.

One of the back analysis methods is to reformulate the governing equations, which describe the relationship between stresses, strain, and rock properties, such that the material properties are treated as unknowns and the measured rock responses the knowns. These reformulated equations are solved directly (i.e., direct inversion approach) (e.g., [40,39,38,32] for the material properties. Simplifying assumptions have been made in order to acquire the solution, including homogeneous material, uniform or linear geologic stress field and one-step excavation [7]. As a result, the estimates are of great uncertainty for real-world problems associated with multi-scale heterogeneity.

Another method is the optimization inversion approach, which automatically adjusts the material property values such that the simulated system responses match the observed in the least square sense. Generally speaking, the adjustment of the parameters to minimize the least square function relies on a search algorithm, such as the Levenberg–Marquardt or gradient methods [3,11,22] or an evolutionary algorithm [18,17]. This approach is different from the direct inversion approach, but it is also built on the homogeneous or zonation parameter field assumption [26,25,24,58,55,7,34], which do not yield detailed descriptions of the spatial variability of the parameters in the field and the uncertainty associated with the estimate.

Besides the aforementioned approaches, probabilistic approaches have been introduced in the past to deal with uncertainty in the parameters due to measurement errors or lack of measurements. For example, using the homogeneous assumption, Ledesma et al. [23] adopted a maximum likelihood approach to identify the elastic modulus and coefficient of lateral earth pressure from measured displacements. Miro et al. [33] conducted the Bayesian back analysis technique to estimate four types of mechanical parameters in a homogeneous rock mass. With a zonation assumption, Sousa and Einstein [46] proposed a Bayesian geologic prediction model to predict stratigraphic distribution using tunnel boring machine performance data and applied to a field case. More recently, Nguyen and Nestorović [35] applied a nonlinear Kalman filter to calibrate tunneling-induced settlements and horizontal displacements for inversing three types of constitutive parameters in a homogeneous case. These back analyses, similar to previously discussed approaches, do not consider the spatial variability of parameters.

Geostatistical theory (i.e., kriging), which considers the spatial

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structure (variogram or covariance) of parameters to be estimated, recently has been employed to derive spatially distributed mechanical parameter fields using available point samples from boreholes (e.g., [30,6,36,8,31]). However, combining the displacements data during excavation and the geostatistical theory as the back analysis has not been reported.

In this paper, we, therefore, introduce a stochastic estimation approach to exploit the displacement data during various excavation stages and spatial statistics of the parameters for the back analysis. A similar concept has been widely used in the hydraulic tomography (HT) for mapping hydraulic properties in the subsurface [51,59]. HT sequentially conducts a series of aquifer hydraulic tests (HT survey), and the resulting potentiometric head changes over a well network are monitored during each test. In essence, the measured responses during each pumping test are tantamount to a snapshot of the aquifer heterogeneity at a selected location. In another word, the HT survey is a hydraulic scan of the subsurface. The complete dataset of observed responses at multiple locations during the HT survey is then jointly analyzed through a stochastic estimation model—successive linear estimator (SLE) ([54,53]). Because of these reasons, HT reveals the detailed spatial distribution of hydraulic properties of the aquifer, patterns of connectivity of highly conductive zones, locations of low conductive zones, and the uncertainties associated with the spatial distribution.

Our proposed sequential collection of displacement data during the successive excavation of a URC is similar to the HT survey. In this paper, we thus adopt the successive linear estimator (SLE) for the back analysis. SLE is an iterative stochastic linear estimator that combines geostatistics and a numerical physical model. Therefore, in the following sections, we discuss the mechanical numerical model first, and then the stochastic conceptualization of heterogeneity. Based on the stochastic conceptualization, a synthetic, two-dimensional numerical model for a URC with spatially varying Young's modulus (E) is created, and the importance of the effect of heterogeneity of rock mass on deformability and stability of the URC is demonstrated. Subsequently, we introduce our stochastic approach, based on SLE, to estimate heterogeneous E field by fusion of the observational displacement data from sequential excavation events. We then discuss the results of our approach for E estimation and its associated uncertainty for each excavation events. Finally, the estimated E fields are evaluated in terms of their abilities to predict displacements and maximum shear strain distribution induced by different excavation events.

2. Mechanical model

In this study, a two-dimensional finite element model is employed for simulating the stress and strain development in a heterogeneous rock mass during the excavation of an unlined rock cavern (see Fig. 1). Consider a two-dimensional cross-section of a dry rock mass (i.e., effects of water pressure are negligible). The stress equilibrium equation under the gravity only can be expressed as:

$$\begin{aligned} &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \\ &\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \gamma_g = 0 \end{aligned} \tag{1}$$

where x represents the horizontal direction and is positive from left to right direction, and y represents the vertical direction and is positive upward. σ_x, σ_y and τ_{xy} are the effective normal stress in x, and y directions, and shear stress, respectively. γ_g is the rock unit weight. Note that this paper defines compressive stress as a negative quantity, while tensile stress a positive quantity.

A linear elastic constitutive law is used to describe the mechanical characteristics of the weathered rock masses where the material constants are Young's modulus and Poisson's ratio. Based on plain strain assumption, the relationship between stress and strain can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E(1-\upsilon)}{(1+\upsilon)(1-2\upsilon)} \begin{bmatrix} 1 & \frac{\upsilon}{1-\upsilon} & 0 \\ \frac{\upsilon}{1-\upsilon} & 1 & 0 \\ 0 & 0 & \frac{1-2\upsilon}{2(1-\upsilon)} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2}$$

where E is Young's modulus, υ is Poisson's ratio and ε_x, ε_y and γ_{xy} are the strain components. On the other hand, the relationship between the strain and displacement is

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \tag{3}$$

where u_x and u_y are the components of displacement in the x- and y-directions. An application of Galerkin's method [47] to Eqs. (1)–(3) yields the following matrix equation

$$[\mathbf{K}_m]\{\mathbf{u}\} = \{\mathbf{f}\} \tag{4}$$

where $\{\mathbf{u}\}$ and $\{\mathbf{f}\}$ are the nodal displacements and nodal force components. For the finite elements in the context of plane elasticity, the element stiffness matrix $[\mathbf{K}_m]$ can be given by integrals of the form

$$[\mathbf{K}_m] = \iint [\mathbf{B}]^T[\mathbf{D}][\mathbf{B}]dxdy \tag{5}$$

where $[\mathbf{B}]$ is the geometry matrix relating strains and nodal displacements (i.e., derived from Eq. (3)), and $[\mathbf{D}]$ contains the constitutive law as a relationship between stresses and strains (i.e., derived from Eq. (2)).

Excavations are represented in the model by assigning a zero value for Young's modulus in the element stiffness matrix at the excavated location. If we denote the elements to be excavated as A, and the stress of these elements before excavation is $\{\sigma_{A0}\}$, the forces acting on the cavern boundary after excavation will be the sum of $\{\sigma_{A0}\}$ and on the self-weight of the elements [42]. That is,

$$\mathbf{f} = \int_{V_A} [\mathbf{B}]^T\{\boldsymbol{\sigma}_{A0}\}dV_A + \gamma \int_{V_A} [\mathbf{N}]^T dV_A \tag{6}$$

where V_A is the excavated volume; $[\mathbf{N}]$ is the element shape functions, which is corresponding to the typical 4-node quadrilateral element.

The initial horizontal stresses (σ_h) are calculated by multiplying the vertical stress (σ_v) with a constant coefficient of lateral earth pressure (k_0), i.e., $\sigma_h = k_0\sigma_v$, while the initial vertical stress (σ_v) determined by its self-weight. No displacement in the x-direction condition (i.e., $u_x = 0$) is assigned to the left-hand side and the right-hand site boundaries while the displacements of the domain bottom are set to be zero (i.e., $u_x = u_y = 0$.)

Moreover, in the mathematical formulation, the following assumptions are made:

(1) The excavation does not alter situ rock properties. That is, the parameter field is assumed to be fixed in time and space during the excavation. This assumption may not be as realistic as it should be. It is, however, convenient for the following analysis since our knowledge of the metamorphosis of the parameter is limited. Nevertheless, our proposed inversion algorithm is not affected by this assumption since it relies on the observed displacement data and prior spatial statistics of the parameter only. In other words, as this approach is applied to a real-world situation, the estimated field will reflect the metamorphosis of the parameter field due to excavation accordingly.
(2) Poisson's ratio (υ) and unit weight (γ_g) are assumed to be constant since they do not have much influence on the back analysis [38]. In addition, for the reason of the difficulty of determining the coefficient of lateral earth pressure (k_0), a reasonable value is assumed and regarded as homogeneous in the domain.