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Research Paper

A numerical homogenization study of the elastic property of a soil-rock mixture using random mesostructure generation

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A R T I C L E I N F O

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A B S T R A C T

A random generation method of periodic mesostructures of soil-rock mixtures based on random polygon is proposed and used in the generation of an RVE model. A code for generating a periodic mesh and periodic boundary is developed for the numerical simulation. Macroscopic elastic parameters of the SRMs are calculated based on the simulation using numerical homogenization. The elastic modulus shows a size effect, namely, the modulus deceases gradually with an increase in the model size. The border effect is related to the model size and is large in small-sized models. Rock aggregates arranged in a certain direction induce property anisotropy.

1. Introduction

An SRM is a common geomaterial widely distributed in the southwest region of China that exhibits rich hydropower resources. Often encountered in huge volumes in some large-scale hydraulic engineering projects, SRMs present a ground instability risk during project construction and operation. For example, large-scale landslides of SRMs have occurred at the Liangjiaren (1.35×10^6 m^3)[1] and Tangjiashan (2.04×10^7 m^3)[2] hydropower station construction sites. These SRM landslides can cause severe damage to hydroelectric projects and endanger human lives. Therefore, the study of the mechanical properties of SRMs is very important for the safe construction of hydropower engineering structures with SRM geomaterials nearby.

A SRM is a highly heterogeneous medium composed of rock and soil (Fig. 1). Many studies on the mechanical properties of SRMs indicate that the mesostructure of an SRM is complicated, and features of the mesostructure such as the fraction and distribution of the rock aggregate play an important role in defining the macromechanical properties of the SRM [3–6]. Due to the size-dependent characteristics of the material, it is difficult and time-consuming to carry out in situ and laboratory experiments at a scale relevant to an engineering design [6]. Fortunately, numerical analysis provides an efficient way to analyze the physical and mechanical behaviors of heterogeneous geomaterials [7]. However, generating a model that is analogous with reality is the prerequisite of successful numerical simulations.

As for the numerical modeling of an SRM, the emphasis is on the characteristics of the rock aggregate, and there are two main methods to build SRM models. One method is a digital image processing-based numerical analysis. This method first obtains images of an SRM through techniques such as photography, CT or X-ray imaging methods, and analyzes the images using a digital image processing (DIP) method. Afterwards, numerical models are built based on the processed images. For example, Yue et al. [8] proposed a method for the vectorization of a bitmap of geotechnical materials. Based on the images obtained by the DIP method, Xu et al. [4,9] studied the strength of an SRM by a finite element method (FEM) and a discrete element method (DEM). Yan and Meng [10] proposed a connected-component-labeling-based DIP algorithm and studied the seepage properties of the SRM.

Another efficient and powerful method is computer simulation using randomly generated mesostructures of the heterogeneous media. Wang et al. [11] used this method to simulate the deformation behavior of concrete. Caballero et al. [12] extended the method from 2D to 3D concrete models. Recently, Xu et al. [2,9] employed this method for SRM and developed a general program for both 2D and 3D problems. Tsesarsky et al. [13] analyzed the elastic moduli and anisotropy of bimrocks (blocks-in-matrix rocks) using a computational homogenization method with a simplification of the rock blocks.

Despite the progress made to date, there is a clear boundary between the macro- and mesoscopic length scales, and bridging this scale range is important for some engineering applications of the numerical

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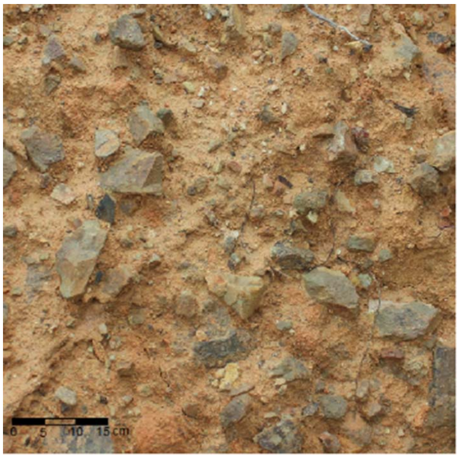

Fig. 1. Typical mesostructure of an SRM.

methods. For hydraulic engineering projects, the macroscopic scale is approximately several kilometers, while the mesoscopic scale is in decimeters. Most studies are based on a single scale analysis and neglect multiscale properties. Traditional stress and strain boundary conditions cannot satisfy both the stress continuity and deformation compatibility conditions from a two-scale perspective. In this situation, numerical homogenization is an efficient way to determine the effective macroscopic properties of an SRM, and the theory and applications of the numerical homogenization method can be found in Hassani and Hinton [14–16].

Our study first proposes a novel random model generation method for SRMs with periodic mesostructures. An automatic program is developed, and methods for periodic mesh generation and periodic boundary condition (PBC) implementation are proposed and implemented. A parametric study of the elastic parameters of an SRM is conducted using the numerical homogenization method. The influence of model size, rock aggregate orientation and fraction on the macroscopic elastic parameters of SRM is investigated.

2. Theory of homogenization

2.1. Asymptotic expansion theory

The idea of homogenization is that a heterogeneous medium with a complex mesostructure can be replaced by an equivalent homogeneous material (Fig. 2). Homogenization relies on an asymptotic expansion of

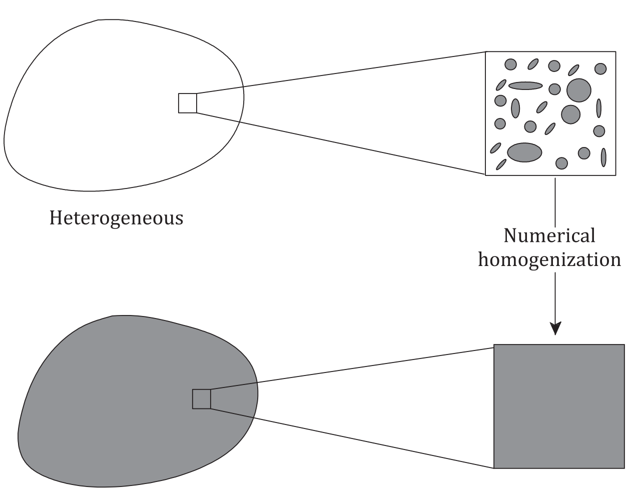

Fig. 2. Illustration of numerical homogenization.

the governing equations, which allows for a separation of scales [17]. Wang et al. [18] proposed a simplified analytical homogenization method to model the behavior of a mixed soil. Currently, homogenization theory has become a mature theory and is widely used in estimating the macroscopic properties of heterogeneous materials [19].

Referring to Yuan and Fish [20], a typical two-scale asymptotic expansion of the displacement field **u** can be written as:

$$u_i^\xi = u_i^0(\mathbf{x},\mathbf{y}) + \xi^1 u_i^1(\mathbf{x},\mathbf{y}) + \xi^2 u_i^2(\mathbf{x},\mathbf{y}) + O(h) \tag{1}$$

where u_i^ξ is the displacement, subscript i indicates the dimension and superscripts 0, 1, and 2 represent the order of differentiation. $\mathbf{x}$ is a position vector in the macroscopic coordinate system, and $\mathbf{y}$ is a position vector with Y-periodicity in the microscopic coordinate system. The two scales are related through a scale factor ξ ($0 < \xi \ll 1$) with $\mathbf{x} = \mathbf{y}/\xi$ and $O(h)$ is a higher-order small value.

Inserting the asymptotic expansion of displacement field into the strong form of boundary value problem (BVP) for the linear elastostatics, two uncoupled problems can be decomposed. The macroscopic BVP finds the solution of the macroscopic displacement u_i^c on domain Ω with surface Γ_u such that:

$$\bar{L}_{ijmn}\bar{\varepsilon}^c_{mn,xj} + \bar{b}_i = 0 \text{on} \Omega \tag{2}$$

$$u_i^c = \bar{u}_i \text{ on } \Gamma_u \tag{3}$$

$$\bar{\sigma}_{ij} n_j = \bar{t}_i \text{ on } \Gamma_t \tag{4}$$

where $\bar{L}_{ijmn}$ is the equivalent material matrix, subscript x_j means the gradient operation in the macroscopic coordinate system and $\bar{\sigma}_{ij}$, $\bar{b}_i$, $\bar{u}$, and $\bar{t}_i$ are the macroscale stress, body force, prescribed boundary displacement, and traction, respectively. $\bar{\varepsilon}_{lm}$ is the macroscale strain component that is defined as:

$$\varepsilon^c_{mn} = \frac{1}{2}\left(\frac{\partial u_m^c}{\partial x_n} + \frac{\partial u_n^c}{\partial x_m}\right) \tag{5}$$

The microscopic BVP finds the solution of the influence function $\chi_{imn}(\mathbf{y})$ on Θ such that:

$$[L_{ijkl}(\chi_{(k,yl)mn} + I_{klmn})]_{yj} = 0 \text{ on } \Theta \tag{6}$$

$$\chi_{imn}(\mathbf{y}) = \chi_{imn}(\mathbf{y} + \mathbf{Y}) \text{ on } \partial\Theta \tag{7}$$

$$\chi_{imn}(\mathbf{y}) = 0 \text{ on } \partial\Theta^{\text{vert}} \tag{8}$$

where

$$I_{klmn} = (\delta_{mk}\delta_{nl} + \delta_{nk}\delta_{ml})/2 \tag{9}$$

$$\chi_{(k,yl)mn} = \frac{1}{2}\left(\frac{\partial \chi_{kmn}}{\partial yl} + \frac{\partial \chi_{lmn}}{\partial yk}\right) \tag{10}$$

and Θ is the domain of the unit cell, $\partial\Theta$, which indicates the domain boundary and $\partial\Theta^{\text{vert}}$ are the domain vertices.The equivalent material matrix can be obtained by volumetrically averaging the numerical solutions of the unit cells and is given as:

$$\bar{L}_{ijmn} = \frac{1}{|\Theta|}\int_\Theta \sigma_{ij}^{mn} d\Theta \tag{11}$$

where σ_{ij}^{mn} are the microscale stresses induced by applying a unit periodic strain ε^c_{mn} in Eq. (5):

$$\sigma_{ij}^{mn} = L_{ijkl}(\chi_{(k,yl)mn} + I_{klmn}) \tag{12}$$

2.2. Numerical implementation

It is seen from the above discussion that the homogenized material property is estimated through the volume average of the numerical analysis result of a unit cell. A unit cell is required to be a representative volume element (RVE). In addition, it is noted that the boundary condition of the unit cell is periodic, and a general expression of the