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Research Paper

Effective stress-based upper bound limit analysis of unsaturated soils using the weak form quadrature element method

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ABSTRACT

Limit analysis methods can be used to directly calculate the bearing capacities of earth structures. Currently, there are very few studies on the limit analysis of unsaturated soils, which are widely distributed in geotechnical engineering. In the present paper, a kinematic theorem of unsaturated soils is established with a unified effective stress employed to represent the matric suction relied shear strength. Numerical solutions of the formulation are obtained by the weak form quadrature element method in combination with the second-order cone programming. Some tests are described, and the effect of suction stress is discussed in detail.

1. Introduction

The determination of the load-carrying capacity of structures remains one of the most crucial issues in civil and geotechnical engineering. There are several ways of performing geotechnical stability analysis and among those methods, limit analysis is a direct means that is based on the fundamental theorems of plasticity [1,2]. Strict upper and lower bounds on the collapse load multipliers can be obtained by applying the kinematic and the static principles of limit analysis. Because of the complexity of actual problems, analytical solutions are barely available, and numerical limit analysis has gained considerable attention in the past four decades [3–5]. In computational limit analysis, the velocity or the stress is discretised spatially in the first step, and then, the resulting optimisation problem is solved by mathematical programming techniques.

In the past few years, numerical limit analysis has been applied to study the stability of a wide range of geotechnical problems, including slopes [6–10], tunnels [11–13], foundations [14–17], and anchors [18–20]. However, very few works have considered the effect of soil saturation on the safety performance of earth structures. Unsaturated soils are frequently encountered in practical engineering, and climate changes, such as the rise in sea level, and rainfall infiltration would lead to the weakening of the shear strength of unsaturated soils. Therefore, it is of great importance to introduce the basic concepts of unsaturated soil mechanics into limit analysis.

One of the important issues in unsaturated soil mechanics is the definition of the basic stress variables for the three-phase material [21]. Most of the available algorithms make use of the independent stress state variables of the net stress and the matric suction to evaluate the shear strength of unsaturated soils [22]. However, the additional angle introduced in that approach to quantify the effect of matric suction is found to be highly dependent on the matric suction and could be negative, which makes it difficult for practical application [23]. The extension of the concept of effective stress to unsaturated soils is a more concise and effective way, especially for the description of the strength of unsaturated soils. Some forms of effective stress have been proposed and validated using field and laboratory test results. A unique relationship was proposed by Khalili et al. [24,25] between the effective stress parameter and the ratio of suction over the air entry value, which has been applied to study some practical problems in geotechnical engineering [26–28]. Recently, the suction stress characteristic curve (SSCC) was proposed by Lu et al. [29,30] by which the shear strength of unsaturated soils can be effectively described by the classical effective stress parameters. The concept of suction stress has been used to compute the thrust of active earth pressures and the stability numbers of slopes by the limit equilibrium method [31,32]. The impacts of rainfall infiltration, the height of the slope and other interested parameters were discussed in detail. However, one-dimensional unsaturated flow conditions were assumed. Furthermore, the drawbacks of the limit equilibrium method are clear: the failure surface must be guessed in advance, and it is very difficult to be generalised to three dimensions [5]. In this study, a procedure is proposed for the limit analysis of unsaturated soils. Although any form of unified effective stress can be incorporated, the suction stress-based effective stress proposed by Lu et al. [29,30] is adopted in this study. A dual formulation of the upper bound theorem is established by incorporating the suction stress into the equations of virtual work. On the basis of the proposed formulation, the spatial discretisation techniques can be combined to preform

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numerical limit analysis of unsaturated soils.

The finite element method has always been used to obtain the numerical solutions of limit analysis problems. In finite element limit analysis method, the continuum is discretised by a number of triangular or tetrahedral elements, where the stress or the velocity is assumed to vary linearly or quadratically. To improve the numerical efficiency and to overcome the volumetric locking, which occurred in incompressible materials, a high-order algorithm: namely the weak form quadrature element method (QEM) is employed in the present study. Compared with the finite element method, the QEM has the characteristic of global approximation and enjoys rapid convergence [33,34]. What is more, integrals in the weak form description of a problem in the QEM are first evaluated by numerical integration, and then the derivatives at an integration point are represented by the differential quadrature analogue. The essence of the differential quadrature analogue is that a derivative is approximated by the linear weighted sum of variables at all sampling points. Thus, algebraic equations are established from which all the function variables are obtained.

A mathematical programming problem arises after spatial discretisation. The optimization problem is formulated as a standard second-order cone programming (SOCP) problem and solved by the optimisation toolbox Mosek. In addition, to obtain the profiles of suction stress, an algorithm and the corresponding Fortran program of unsaturated steady seepage analysis are established using the QEM. For convenience, identical mesh grids are utilised for seepage and limit analysis and consequently the nodal pore pressures obtained from seepage analysis can be directly used without any additional work.

The rest of this paper is organised as follows. In Section 2, the unified effective stress and the formulation of upper bound limit analysis of unsaturated soils are described in detail. Section 3 gives the procedure of spatial discretisation by the QEM. The SOCP and concise information on unsaturated seepage analysis are provided in Sections 4 and 5, respectively. The numerical tests regarding unsaturated slopes and foundations are given in Section 6, and the concluding remarks are provided in Section 7.

2. Upper bound theorem of unsaturated soils

2.1. Suction stress-based effective stress

The unified effective stress for saturated and unsaturated soils based on the suction stress proposed by Lu et al. [29] is employed here to represent the contribution of matric suction to the shear strength of unsaturated soils. Only the classical effective strength parameters are needed, and we don't need to introduce additional parameters. The effective stress can be expressed in vectorial form as follows:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + u_a\mathbf{I} + \sigma^s\mathbf{I} \tag{1}$$

where $\boldsymbol{\sigma}'$ is the effective stress, $\boldsymbol{\sigma}$ is the total stress, u_a is the pore air pressure, $\mathbf{I} = [1\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0]^{\mathrm{T}}$ is an identity vector, and σ^s is the suction stress. Suction stress represents the active forces at or near interparticle contacts, which do not propagate from one soil grain to another through the granular skeleton [29]. A closed form solution of suction stress has been proposed as follows [30]:

$$\begin{aligned}&\sigma^s = -(u_a - u_w) \qquad (u_a - u_w) \leqslant 0\\ &\sigma^s = -\frac{(u_a - u_w)}{\{1 + [\alpha(u_a - u_w)]^n\}^{(n-1)/n}} \quad (u_a - u_w) > 0\end{aligned} \tag{2a-b}$$

where u_{w} is the pore water pressure. α and n are the fitting parameters of the soil water characteristic curve presented by van Genuchten and Mualem [35], where α is the inverse of the air entry pressure and n is the pore size distribution parameter.

2.2. Upper bound theorem for single-phase material

Suppose that a rigid perfectly plastic body Ω is subjected to a body force $\lambda\mathbf{g}$ throughout the domain and a surface force $\lambda\mathbf{T}$ on Γ_s, where $\mathbf{g}$ and $\mathbf{T}$ are specified beforehand and λ is the load multiplier that is to be optimised. The whole boundary can be divided into $\partial\Omega = \Gamma_u \cup \Gamma_s$, $\Gamma_u \cap \Gamma_s = \varnothing$, where Γ_u is the portion of boundary where the velocity is prescribed. The upper bound theorem states that, if a kinematically and plastically admissible velocity state could be found, which satisfies the boundary conditions and the plastic flow rule, an upper bound on the limit load multiplier could be predicted by equating the power expended by the external loads to that dissipated by plastic deformation. The mathematical programming problem can be expressed as follows:

$$\begin{aligned}&\lambda = \text{minimise } W_{\text{int}} = \text{minimise } \int_\Omega D(\dot{\boldsymbol{\varepsilon}})\mathrm{d}V\\ &\text{s.t. } W_{ext} = \int_\Omega \mathbf{g}^{\mathrm{T}}\dot{\mathbf{u}}\mathrm{d}V + \int_{\Gamma_s}\mathbf{T}^{\mathrm{T}}\dot{\mathbf{u}}\mathrm{d}S = 1\\ &\dot{\boldsymbol{\varepsilon}} = \mathbf{L}\dot{\mathbf{u}} \quad \text{in } \Omega\\ &\dot{\mathbf{u}} = \mathbf{0} \text{ on } \Gamma_u\end{aligned} \tag{3a-d}$$

where W_{int} is the internal work rate, W_{ext} is the external work rate, $D(\dot{\boldsymbol{\varepsilon}})$ is the plastic dissipation function, and $\dot{\mathbf{u}}$ and $\dot{\boldsymbol{\varepsilon}}$ are the velocity and the strain rate, respectively, which can be related by a differential operator $\mathbf{L}$. The dual formulations of Eq. (3a-d), also known as the stress-based upper bound problem [36], are given as follows:

$$\begin{aligned}&\text{maximise} \quad \lambda\\ &\text{s.t.} \qquad \int_\Omega \delta\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}}\boldsymbol{\sigma}\mathrm{d}V = \lambda\left(\int_\Omega \delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{g}\mathrm{d}V + \int_{\Gamma_s}\delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{T}\mathrm{d}S\right)\\ &\qquad\quad f(\boldsymbol{\sigma}) \leqslant 0\end{aligned} \tag{4a-c}$$

where $f(\boldsymbol{\sigma})$ is the yield function. It can be noticed that, Eq. (4b) represents the equation of virtual work in elasto-plastic analysis. As discussed by Krabbenhoft et al. [36] and Makrodimopoulos and Martin [37], dual formulations are preferable for the following reasons: less number of free variables is required, linearly dependent equality constraints are less likely to appear, and the inclusion of discontinuities is straightforward. Therefore, the dual formulations are employed in the present paper.

2.3. Upper bound theorem for unsaturated soils

After the introduction of the unified effective stress Eq. (1), the shear strength or yielding condition of unsaturated soils can be solely defined as a function of the effective stress. Thus, the virtual work equation Eq. (4b) should be also expressed in terms of the effective stress, and to achieve this, we incorporate Eq. (1) into Eq. (4b) as follows:

$$\begin{aligned}&\int_\Omega \delta\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}}\boldsymbol{\sigma}\mathrm{d}V = \int_\Omega \delta\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}}(\boldsymbol{\sigma}' - u_a\mathbf{I} - \sigma^s\mathbf{I})\mathrm{d}V = \lambda\left(\int_\Omega \delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{g}\mathrm{d}V + \int_{\Gamma_s}\delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{T}\mathrm{d}S\right)\\ &\Rightarrow \int_\Omega \delta\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}}\boldsymbol{\sigma}'\mathrm{d}V = \lambda\left(\int_\Omega \delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{g}\mathrm{d}V + \int_{\Gamma_s}\delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{T}\mathrm{d}S\right) + \int_\Omega \delta\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}}(u_a\mathbf{I} + \sigma^s\mathbf{I})\mathrm{d}V\\ &\Rightarrow \int_\Omega \delta\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}}\boldsymbol{\sigma}'\mathrm{d}V = \lambda\left(\int_\Omega \delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{g}\mathrm{d}V + \int_{\Gamma_s}\delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{T}\mathrm{d}S\right) + \int_\Omega \delta\dot{\varepsilon}_{\mathrm{v}}u_a\mathrm{d}V\\ &\quad + \int_\Omega \delta\dot{\varepsilon}_{\mathrm{v}}\sigma^s\mathrm{d}V\end{aligned} \tag{5a-d}$$

The upper bound formulation of unsaturated soils can then be summarised as follows:

$$\begin{aligned}&\text{maximise} \quad \lambda\\ &\text{s.t.} \qquad \int_\Omega \delta\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}}\boldsymbol{\sigma}'\mathrm{d}V = \lambda\left(\int_\Omega \delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{g}\mathrm{d}V + \int_{\Gamma_s}\delta\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{T}\mathrm{d}S\right)\\ &\qquad\qquad + \int_\Omega \delta\dot{\varepsilon}_{\mathrm{v}}(u_a + \sigma^s)\mathrm{d}V\\ &\qquad\quad f(\boldsymbol{\sigma}') \leqslant 0\end{aligned} \tag{6a-c}$$

If saturated soils are considered, Eq. (6) becomes

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