Computers and Geotechnics 97 (2018) 1–6

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Technical Communication

Buckling of tapered friction piles in inhomogeneous soil

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ARTICLE INFO

Keywords:
Tapered pile
Shaft friction
Buckling
Buckled shape
Soil-structure interaction

ABSTRACT

In this paper, an analytical model is developed to estimate the buckling behavior of tapered friction piles fully embedded in inhomogeneous soil. The governing differential equation of the problem is derived with associated boundary conditions and is solved by using the Runge-Kutta method in combination with the Regula-Falsi method. Numerical examples for calculated buckling loads and buckled shapes are given to highlight the introduction of dimensionless variables related to the tapering and shaft friction of the pile, soil inhomogeneity and pile-soil stiffness as well as the degrees of freedom at the both ends of the pile.

1. Introduction

The buckling instability of long slender piles particularly in soft soils is a key consideration in geoengineering design. Potential buckling failure exists when slender piles are embedded in soft soil, erodible soil and liquefiable soil [1–4]. Furthermore, with the ongoing evolution of pile applications to include higher capacity (i.e., higher allowable stress) of the pile cross section the common design practices that fully embedded piles will not buckle before yielding of the pile cross section is no longer valid [2].

Extensive studies have been performed on the buckling response of axially loaded piles. An early approach for the stability of beams on elastic foundations by Hetenyi [5] may be extended to the buckling analysis of piles supported laterally by elastic foundations. For example, Bjerrum [6] derived exact solutions for the buckling load of piles pinned top and bottom. He also compared the calculated buckling loads to test data from pile load tests in soft clay. Davisson and his coworker [7,8] investigated the effects of partial embedment and different degrees of freedom at the ends of the pile on the buckling behavior. Prakash [9] used the energy method to compute the buckling capacity of piles. West et al. [10] presented buckling loads of piles and the corresponding clustering pattern of buckling modes. By considering the nonlinear lateral soil support (bi-linear p-y curves), Vogt et al. [2] evaluated the inelastic buckling of pinned-pinned piles and validated the obtained results by comparing with model test data as well as results from loading tests on 4 m long slender piles supported by soft clay. Chen et al. [11] employed the cusp catastrophe theory to construct the mathematical model for assessing the buckling load of piles. Deng et al. [12] derived an equation for the instability of piles supported by the modified Vlasov foundation model and provided numerical buckling solutions by the vibrational approach. Recently, Lee [13] elucidated the influences of tapering and cross section shape on the buckling of piles whose volumes are always held in constant. In the abovementioned analyses, however, it was assumed that the axial load is constant along the pile, that is, no load transfer occurs throughout the pile shaft and which is thus applicable for relatively short stubby end-bearing piles. It seems likely that the shaft resistance along the pile affects the buckling of the friction piles. There only exists a few solutions that take into account shaft friction of the pile-soil system. These have been done for straight friction piles by using the energy method [14], Rayleigh-Ritz method [15] and modal clustering technique [16]. However, to the best knowledge of the authors, the buckling of tapered friction piles has not been studied in the open literature.

The aim of this study is to introduce an analytical model to deal with the buckling of tapered piles that take into account both the lateral stiffness and shaft friction. The governing equation of fully embedded tapered friction piles with different boundary conditions is derived and solved numerically. The versatility of the proposed approach is illustrated using numerical examples of the pile-soil system for a wide range of values of geometric parameters and material properties. The obtained results for simplified cases of the current problem are compared with available analytical solutions.

2. Formulation

Fig. 1(a) shows a fully embedded vertical pile defined using the Cartesian coordinate system (x, y) with length L in the x direction and radius r in the y direction. The pile is elastic, homogeneous, and circular

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https://doi.org/10.1016/j.compgeo.2017.12.012
Received 1 August 2017; Received in revised form 24 October 2017; Accepted 24 December 2017

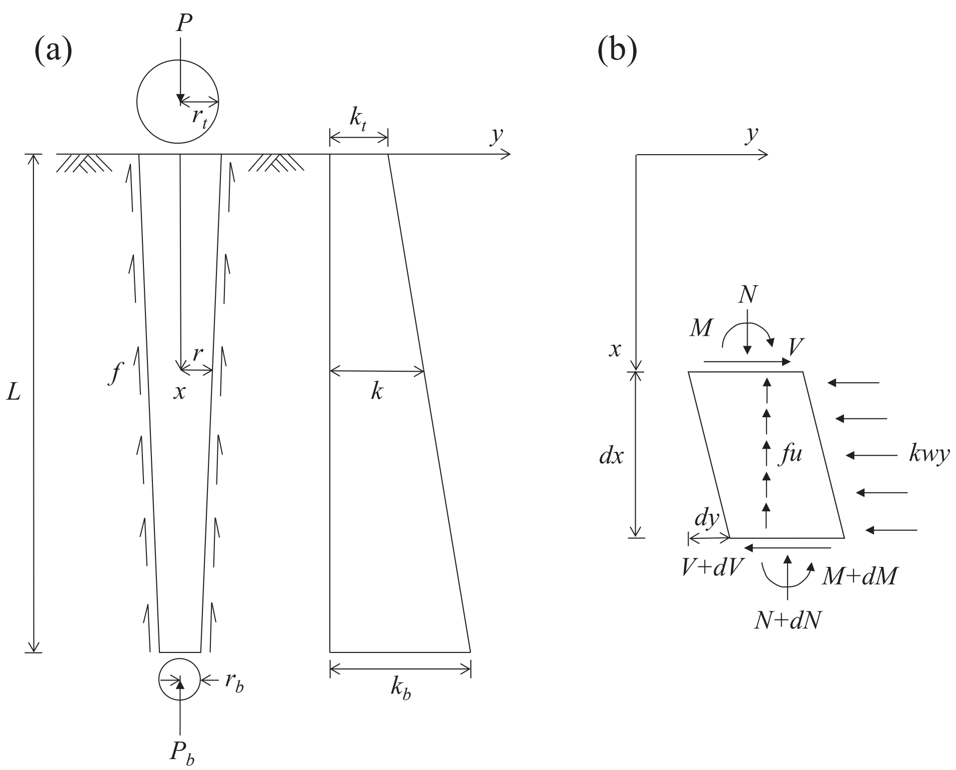

Fig. 1. Schematics of proposed model: (a) tapered friction circular pile; (b) deflection and forces on pile element.

in cross section, but the radius can vary longitudinally. Linear variation is considered for the length of the pile as

$$T_x = \left[1 + (m-1)\frac{x}{L}\right] \tag{1}$$

where m is the taper ratio, defined as

$$m = \frac{r_b}{r_t} \tag{2}$$

in which, r_b and r_t are the radii at the pile base and top, respectively. In this study, the pile is treated to be tapered down with depth, i.e., $0 \le m \le 1$. The pile radius at any depth is given by

$$r = r_t T_x \tag{3}$$

Putting $x = L/2$ in Eq. (1) and using Eq. (3), the radius at the midpoint of the pile length r_e is obtained as

$$r_e = \frac{r_t}{2} m_1 \tag{4}$$

where $m_1 = m + 1$.

Note that owing to the linear taper of the pile, the midpoint radius is equal to the mean pile radius through the embedded length. The diameter w_e, perimeter u_e and moment of inertia I_e of the cross section at the midpoint are expressed as

$$w_e = m_1 r_t; \quad u_e = \pi m_1 r_t; \quad I_e = \frac{\pi}{64} m_1^4 r_t^4 \tag{5}$$

In a similar manner, the diameter w, perimeter u and moment inertia I at any depth are estimated by using Eq. (3) with Eq. (5):

$$w = \frac{2w_e}{m_1} T_x; \quad u = \frac{2u_e}{m_1} T_x; \quad I = \frac{16 I_e}{m_1^4} T_x^4 \tag{6}$$

For a fully embedded pile, the soil lateral stiffness may be represented by an elastic Winkler foundation with a linearly increasing coefficient of subgrade reaction in unit of force per length^3 [17]. The coefficient of subgrade reaction with depth can be written as

$$k = k_t H_x \tag{7}$$

$$H_x = \left[1 + (n-1)\frac{x}{L}\right] \tag{8}$$

where n is the soil inhomogeneity, defined as

$$n = \frac{k_b}{k_t} \tag{9}$$

in which, k_b and k_t are the coefficient of horizontal subgrade reaction at the pile base and top, respectively. The coefficient of subgrade reaction for $x = L/2$ is obtained as

$$k_e = \frac{k_t}{2} n_1 \tag{10}$$

where $n_1 = n + 1$. The combination of Eqs. (7) and (10) allows the coefficient of subgrade reaction profile k to be determined:

$$k = \frac{2k_e}{n_1} H_x \tag{11}$$

As illustrated in Fig. 1(a), the pile is subjected to an axial compressive load P at its top and a reactive force P_b at its base. The interactions between the pile and surrounding soil are denoted by f as side resistance acting on the annular surface at depth per unit length of the pile (known as unit shaft friction). Fig. 1(b) shows the stress resultants of an infinitesimal element of the deformed pile, generated by the applied compressive load. Based on the free body diagram, the ordinary differential equation is derived by satisfying the equilibrium of the pile element:

$$\frac{dN}{dx} + fu = 0 \tag{12}$$

$$\frac{dV}{dx} + kwy = 0 \tag{13}$$

$$\frac{dM}{dx} - V + N\frac{dy}{dx} = 0 \tag{14}$$

where N, V and M are the axial and shear forces and bending moment in the pile, respectively. By using the stress-strain relationship of the pile element, i.e.,