Computers and Geotechnics 98 (2018) 189–196

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

# Upper bound solutions for face stability of circular tunnels in non-homogeneous and anisotropic clays

Maosong Huang[a,b,*], Zhen Tang[a,b], Weixiang Zhou[c], Juyun Yuan[a,b]

[a] Department of Geotechnical Engineering, Tongji University, Shanghai 200092, China
[b] Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Tongji University, Shanghai 200092, China
[c] Shanghai Municipal Engineering Design Institute (Group) Co., Ltd., Shanghai 200092, China

ARTICLE INFO

*Keywords:*
Clays
Tunnel face stability
Three dimensions
Limit analysis

ABSTRACT

This paper presents a three-dimensional stability analysis of a circular tunnel face in non-homogeneous and anisotropic undrained clay using the kinematic approach (upper bound) of limit analysis. The proposed failure mechanism consists of a cylindrical rigid block and a toroidal shear zone with variable radius, and the closed-form analytical expressions of velocity field are derived within the framework of an orthogonal curvilinear coordinate system. The critical collapse-supporting pressure and stability ratio are obtained through optimization with respect to the geometrical parameters of the mechanism. Two types of non-homogeneous undrained strength, linearly changing with depth, and two-layer clays with constant undrained strength are investigated. Meanwhile, the 3-D finite element analysis is employed to validate the proposed failure mechanism. The upper bound solutions of the stability ratio of the proposed mechanism compare reasonably well with the upper bound solutions from the finite element analysis and show significant improvements over the existing upper bound solutions in single layer clays with homogeneous and isotropic undrained strength. The results show that the critical collapse pressure decreases with the increase in the non-homogeneous ratio and the anisotropic ratio and increases with the ratio between the undrained strength of the top layer and of the bottom layer.

## 1. Introduction

In practical tunnelling projects using a pressurized shield, it is of vital importance to determine the adequate range of the face-supporting pressure applied by the shield. If the supporting pressure is not sufficient, the soil will move towards the tunnel face, and a soil collapse may occur. In contrast, if the supporting pressure is too great, the soil is "pushed" towards the ground surface, and then a blowout may appear. Therefore, it is desirable to determine the adequate range of face supporting pressure to prevent both kinds of failure.

Under three-dimensional conditions, the stability of the tunnel face of circular tunnels in purely cohesive soil has been investigated by several authors in the literature. Some authors have adopted the kinematic approach of limit analysis [1–7]. Davis et al. [1] proposed a translational failure mechanism based on two oval-shaped rigid blocks, while only an inscribed elliptical area of the tunnel face was considered in the failure mechanism. The failure mechanisms considering the entire tunnel face were developed by Mollon et al. [2,3], mechanisms in which the translational or rotational movements of the rigid blocks were generated by a spatial discretization technique. Klar et al. [4] suggested a new mechanism with a continuous velocity field based on the use of incompressible flow fields derived from the theory of elasticity and the concept of sinks and sources. The proposed velocity field is proportional to the elastic displacement field, while only partial failure of the tunnel face was involved. Mollon et al. [5] constructed two ingenious toroidal failure mechanisms based on the deformation fields observed in centrifuge tests and numerical analysis of undrained clay. One mechanism is based on a symmetrical velocity field and the other is based on a non-symmetrical velocity field. Those mechanisms involve a continuous velocity field with no discontinuities at the boundaries, and they were computed by a mixed analytical-numerical procedure. Klar and Klein [6] investigated those mechanisms using a closed-form analytical expression, and both the upper-bound calculations of stability and the mobilized strength design (MSD) calculations of volume loss were performed. Moreover, Zhang et al. [7] further investigated the mechanism involving a non-symmetrical velocity field proposed by Mollon et al. [5] with some modifications, and the upper bound analyses were performed using a closed-form expression to investigate the face stability of the circular tunnels in clay with a linearly increased undrained strength.

* Corresponding author at: Department of Geotechnical Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China.
*E-mail address:* mshuang@tongji.edu.cn (M. Huang).

https://doi.org/10.1016/j.compgeo.2018.02.015
Received 14 September 2017; Received in revised form 8 February 2018; Accepted 12 February 2018

Among the works mentioned above, soils are assumed to be materials with isotropic strength. However, due to the preferred particle orientation during sedimentation and the spatial variation of the fabric, as well as the complex and anisotropic stress states of loading or unloading, most natural soils exhibit anisotropic strength characteristics. Yang et al. [8] studied the influence of anisotropic and non-homogeneous strength on the critical supporting pressure of the tunnel face in a *c*-*φ* soil under two-dimensional conditions. Both the anisotropy of the friction angle and cohesion are considered. Pan and Dias [9] studied the three-dimensional face stability of the tunnel face in multi-layered *c*-*φ* soil using the spatial discretization technique, and the anisotropy of soil cohesion is considered. Klar and Elkayam [10] investigated the stability of the tunnel face based on a new asymmetric yield function that allows the shearing strength in the extension mode to be smaller than the shearing strength in the compression mode. The kinematically admissible velocity fields for upper bound analysis were generated numerically. All these studies showed that the anisotropy of strength exerts significant influence on the critical supporting pressure.

For the face stability problems in purely cohesive soil, the influence of anisotropic undrained strength has not so far been investigated. Therefore, this paper focusses on the three-dimensional face stability of a circular tunnel in non-homogeneous and anisotropic undrained clay using the kinematic approach of limit analysis. A mechanism consisting of a rigid block and a distortional shear zone is first presented, different from the failure mechanisms comprised of either rigid blocks or distortional shear zones mentioned above. Within the failure mechanism, the entire circular area of the tunnel face was considered, as well as the soil mass in the distortional shear zone "flow" towards the tunnel face rather than the translation or rotational motion of the rigid blocks. The upper bound calculations were performed within the framework of an orthogonal curvilinear coordinate system. The proposed mechanism is first validated by comparing with the results of the three-dimensional finite element analysis as well as other existing solutions. Then, two types of non-homogeneous undrained strength, linearly changing with depth and two-layer clay with constant undrained strength, are analysed. For both types, the impact of anisotropy of undrained strength on the critical collapse pressure and failure mechanism is considered and discussed.

## 2. Problem statement

A circular tunnel with diameter $D$ excavated under a cover depth $C$ in two-layered clays is considered. Fig. 1 illustrates the geometry and parameters of the idealized problem. The load surcharge $\sigma_T$ on the ground surface and the supporting pressure $\sigma_S$ on the tunnel face are assumed to be uniformly distributed, as shown in Fig. 2. The top layer of clay of thickness $H$ is underlain by a clay layer with infinite depth. The undrained strength of the clay layers can be characterized by the following equation, as

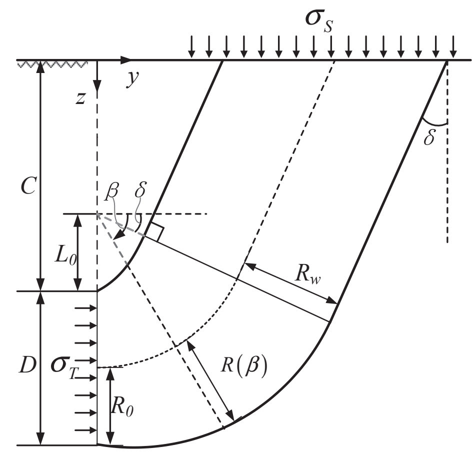


**Fig. 2.** Failure mechanism in plane of symmetry $x = 0$.

$$c_u(z) = \begin{cases} c_{u0-t} + \rho z, & 0 \leqslant z < H \\ c_{u0-b} + \rho(z-H), & H \leqslant z \end{cases} \tag{1}$$

where $c_{u0\text{-}t}$ and $c_{u0\text{-}b}$ are the undrained strength at the ground surface and the top-bottom interface, respectively. $\rho = dc_u/dz$ is the rate of the undrained strength $c_u$ increasing with depth $z$.

The anisotropic shear strength of the soil refers to the values of undrained shear strength changing with the rotation of the largest major principal stress. The anisotropy of the undrained shear strength has been investigated by many researchers [11–13], and the variation of the undrained strength with direction can be described approximately by the curves in Fig. 3. The anisotropic undrained shear strength at the place where the major principal stress is inclined at an angle $i$ to the vertical direction can be expressed as [11,12]

$$c_u(i) = c_{uh} + (c_{uv} - c_{uh})\cos^2 i \tag{2}$$

where $c_{uv}$ and $c_{uh}$ are the vertical and horizontal undrained shear strength, respectively, $i$ is the inclination of major principal stress with vertical direction. For undrained conditions, the relative velocities are parallel to the respective discontinuities and assumed to be inclined at an angle $\pi/4$ to the directions of major principal stresses. Therefore, the anisotropic angle $i$ can be calculated directly from the directions of

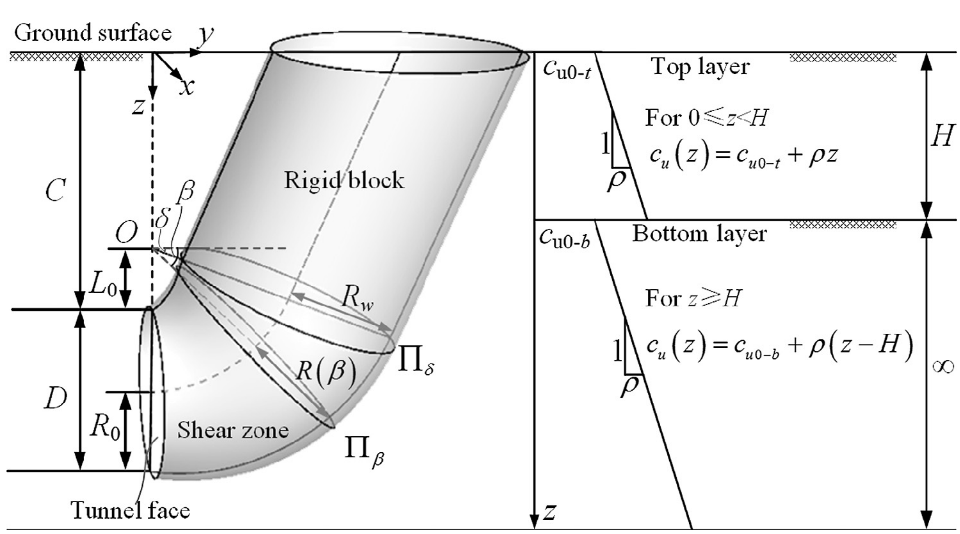


**Fig. 1.** General layout and failure mechanism of the problem.