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Cross-stream forces and velocities of fixed and freely suspended particles in viscoelastic Poiseuille flow: Perturbation and numerical analyses

Eric F. Lee, Donald L. Koch*, Yong Lak Joo

School of Chemical and Biomolecular Engineering, Cornell University, Ithaca, NY 14853, USA

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ABSTRACT

The cross-stream migration of a circular particles (or infinitely long cylinder) in two dimensional, inertia-less viscoelastic pressure-driven flows is examined through complementary finite element simulations and second-order fluid perturbation analyses for small Deborah number (*De*), where *De* is defined as the fluid relaxation time divided by the characteristic flow time. A neutrally buoyant, freely suspended particle migrates toward the center of the channel for all particle sizes and cross-stream positions due to the coupled effects of the linear and quadratic variations of the imposed velocity. A particle that is held at a fixed position, in contrast, experiences a cross-stream force directed toward the wall as a result of the coupled effects of the local shear flow and the flow relative to the particle.

1. Introduction

Particles migrate in viscoelastic suspensions under appropriate conditions to preferred cross-stream positions due to surface forces perpendicular to the fluid streamlines. These forces occur due to inhomogeneous velocity and/or temperature fields that alter fluid properties near the particle. As a result of particle migration, inhomogeneous suspensions form that can have significantly different rheological properties, with consequences for processing applications such as injection molding. These applications have motivated several studies that have looked at the forces acting on a particle suspended in a viscoelastic Poiseuille flow, but the previous literature has not shown a clear result for the parameters that affect the direction and magnitude of the cross-stream force on a particle due to the fluid. No study to date has included an extensive comparison of quantitative simulation and analytical results, which is of great value in a topic with contradictory findings on even the most fundamental qualitative result of the migration direction. In light of this, the current study develops a full 2D theory to predict the cross-stream force and migration velocity for a particle in a plane Poiseuille flow at low *De*, for both the case of a particle moving with the fluid and the case of a particle fixed in place. Here the Deborah number, *De*, is defined as the fluid relaxation time divided by the characteristic flow time. Trajectories for a freely suspended particle at low *De* are computed based on the analytical predictions. Finite element method (FEM) simulations are performed for a circular particle in plane Poiseuille flow of an Oldroyd-B fluid at finite *De*. To facilitate a parametric study examining the effects of *De*, particle size, particle location, and polymer concentration, we consider (a) a particle that is fixed in position; and (b) a freely suspended particle which experiences no streamwise force and no torque but is maintained at a fixed cross-stream position by balancing the cross-stream non-Newtonian surface force with a cross-stream body force. The tendency toward migration is then measured by the magnitude of the cross-stream surface force. The FEM simulations are in good agreement with the theory in the limits of small *De* and small particle size.

Previous theoretical attempts to explain the migration phenomenon have examined the effects of fluid–particle interactions on the stress and velocity fields (e.g. [1–3]). In an inertia-less flow, migration in a dilute suspension is only predicted to occur in non-Newtonian fluids; with these types of suspending fluids, particles are predicted to migrate preferentially to either the wall or the center of a bounded flow, depending on system conditions. Ho and Leal [4] studied the particle migration of a sphere through a small *De* perturbation analysis in a second-order fluid. They derived an expression for the cross-stream migration force on a small particle (radius $\ll$ channel height) in Poiseuille flow, finding that the particle always migrates to the center of the pipe. They found that the migration force is a function of the ratio of the particle size to the

* Corresponding author.
E-mail addresses: ericlee@cornell.edu (E.F. Lee), dlk15@cornell.edu (D.L. Koch).

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channel height, the position of the particle, and the first and second normal stress differences. Ho and Leal [4] also investigated circular Couette flow, finding that particles migrate to the outer wall, where the absolute shear rate is at a minimum.

Several numerical studies have also addressed the particle migration phenomenon. Carew and Townsend [5] found in their finite element simulations that a fixed cylinder in an inertia-less plane Poiseuille flow experiences a cross-stream force toward the wall in both Oldroyd-B and Phan–Thien–Tanner fluids. Huang et al. [6] also performed FEM simulations of a cylindrical particle in plane Poiseuille flow of an Oldroyd-B fluid. They plotted the trajectories of a freely suspended particle released at a point in the fluid with zero initial translational and rotational velocities. Most of their computations examined the coupled effects of viscoelasticity, fluid inertia, and particle inertia. In some simulations, they neglected the nonlinear fluid inertia term in the fluid momentum equation but still retained the transient fluid inertia term and the particle inertia. They found that the direction of the cross-stream migration of the particle depended on the particle size with smaller particles migrating toward the center and larger particles migrating toward the walls. Thus, although two computational studies have examined the cross-stream force on a particle in a low *De* plane Poiseuille flow, there are no previous studies that have modelled freely suspended particles (with zero streamwise force and zero torque) in inertia-less shear flows. The case of inertia-less freely suspended particles is often of greatest interest in processing applications involving high viscosity polymeric fluids, has been most often considered in theoretical and experimental work and is the emphasis of the present study.

No experimental studies to date have exactly mimicked the conditions of the previous analytical and numerical studies. Nevertheless, several investigators have studied particles in creeping, viscoelastic Poiseuille flows, finding contrasting results for the direction of the cross-stream force. Karnis and Mason [7] did some of the first work on a highly concentrated (particle weight fraction of 0.125) suspension of spheres in a shear thinning polymer solution, finding that the spheres migrated to the center of a pipe, regardless of their initial position. More recently, Jefri and Zahed [8] found that spherical particles in a constant viscosity viscoelastic fluid migrated toward the center plane, while particles in shear-thinning viscoelastic fluids migrated toward the plates (or walls) of the channel in a creeping Poiseuille flow. Several other experimental and numerical studies also suggest that shear thinning may promote migration toward the walls in a pressure driven flow of a viscoelastic suspension (see [9,10,3]). Jefri and Zahed used a particle volume fraction of 0.02, so there were some hydrodynamic particle–particle effects in the system, with most particles aggregating into short chains of 2–10 particles. However, even though their study examined multiple particles rather than a single particle, their work on constant viscosity viscoelastic fluids provides a good source of comparison for theory because they studied the appropriate particle size limit and flow regime for the perturbation analysis of Ho and Leal [4]. In contrast to the results of Jefri and Zahed [8], Dhahir and Walters [11] found that a fixed, freely rotating single cylinder felt a cross-stream force toward the channel wall in Poiseuille flow conditions for both Boger and pseudoplastic fluids. The experimental data from Dhahir and Walters [11] for a fixed particle in a suspending Boger fluid will be compared with FEM simulation results obtained in this study.

In summary, the past investigations have come to varied conclusions on the direction of particle migration in a non-Newtonian Poiseuille flow. While perturbation analyses indicate that a small, inertialess, freely suspended particle always experiences a cross-stream force toward the channel center, some simulations and experiments have shown a cross-stream force in the opposite direction. The only explanation for the switch in cross-stream force direction that has been advanced is that particle size is the main parameter affecting the cross-stream force direction. We do not reach the same conclusion and find that in fact the direction of the cross-stream force is related to the boundary condition on the particle.

We hope to resolve the discrepancies in the literature using perturbation and computational analyses of the two-dimensional flow and stress profiles on the surface of the particle and a thorough examination of the effects of flow geometry, fluid parameters, and the particle boundary condition on the particle cross-stream force. Our 2D perturbation analysis will allow for a direct comparison between analytical and numerical work, which until now was not possible because all of the previous theoretical work was performed in 3D while the numerical work was performed in 2D. The methodology of the 2D analysis differs significantly from the prior 3D work by Ho and Leal [4]. The 3D theory for migration of a spherical particle at low Deborah number in a Poiseuille flow required an application of the generalized reciprocal theorem to obtain the cross-stream force without an explicit determination of the perturbed fluid velocity and pressure field. However, particle–wall hydrodynamic reflections could be neglected to leading order for small particle sizes. In contrast, the Tanner–Pipkin theorem allows us to determine the cross-stream force on a circular particle in a two-dimensional flow without application of the generalized reciprocal theorem. The Tanner–Pipkin theorem states that the Newtonian and second-order fluid velocity fields are the same in a two-dimensional flow and it provides a prescription for the perturbed pressure field. In the two-dimensional problem, hydrodynamic particle-wall interactions are important in evaluating the migration velocity of a particle owing to the logarithmic nature of the fluid velocity produced by a translating cylinder.

In the first section, we will outline the system in consideration, a description of the problem, and the methodology for solution. In the next section, we will present the derivation of the expressions for the cross-stream force and velocity for a freely suspended particle along with complementary FEM simulations results. In the last section, we will derive the cross-stream force on a fixed particle and compare the expression to previous experimental results from the literature. The FEM simulations will be used both to test the range of validity of the asymptotic theory and to perform a parametric study that extends beyond the asymptotic limits.

2. Approach

We consider the two-dimensional pressure driven flow of a non-Newtonian fluid in a channel containing one circular particle as illustrated in Fig. 1. In the absence of the particle, the fluid velocity would be parabolic because the constitutive equations we consider do not exhibit shear thinning.

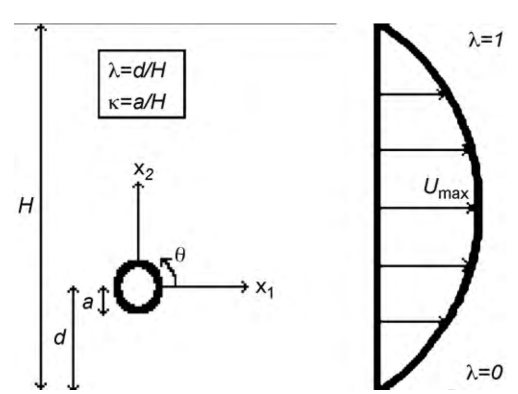

Fig. 1. Schematic of plane Poiseuille flow containing a circular cylinder.