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Research Paper

Influence of model type, bias and input parameter variability on reliability analysis for simple limit states with two load terms

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ABSTRACT

A general closed-form solution to compute the reliability index of a simple limit state function with two load terms and one resistance term is derived. The formulation considers contributions to margins of safety expressed in probabilistic terms due to the choice of load and resistance models, bias values, dependencies between nominal values and bias, uncertainty in estimates of nominal values for uncorrelated load and resistance terms at time of design, and average margin of safety expressed as the operational factor of safety. A sensitivity analysis and example application demonstrate the quantitative influence of the contributing random variables on reliability index.

1. Introduction

In this paper we focus on a particular class of simple linear limit state functions with two load terms and one resistance term. This paper continues the work of Bathurst and Javankhoshdel [1] who restricted their study to the case of one resistance term and only one load term and developed a closed-form solution for the reliability index for performance functions of this type. The motivation for the work is reliability-based design of geotechnical soil and soil-structure problems using limit state (performance) functions that have uncertainty due to the accuracy of the underlying equations that appear in a limit state equation, accuracy of the calibration of limit state equations that have an empirical component, and variability in the selection of nominal values at time of design. Performance functions of this type are common in geotechnical soil-structure interaction problems (e.g. [2–4]). Some examples are pullout and rupture limit states in mechanically stabilized earth (MSE) walls [5,6], soil nail walls [7] and compression piles [8]. For MSE walls and soil nail walls the first load contribution is the permanent load due to soil self-weight and the second load is the result of a permanent surface load (such as a footing), or possibly additional load due to an extreme event such as earthquake. For compression piles, the first load term would be associated with structure dead loads and the second could be due to sustained live loads [9].

2. Objective and scope

The key objective in the current study is the derivation of a general expression for reliability index (β) that captures quantitatively the accuracy of the underlying load and resistance equations that appear in a simple limit state function (called method error or bias) and uncertainty in the choice of nominal input values used at design time. The former is the combined effect of underlying model error, or model bias, and uncertainty in the back-fitting of coefficient terms that may be present in the models at the time of model calibration.

The load and resistance terms in the current and earlier study [1] are uncorrelated but both include random variables that capture method bias in the estimate of the nominal resistance value and bias in each nominal load contribution. Method bias is computed as the ratio of measured (actual) load or resistance value to the corresponding predicted (nominal) load or resistance value used at the time of design. Bias values can be understood to be corrections applied to nominal load and resistance values that are prescribed or calculated using the underlying load and resistance models that appear in the performance function of interest. These corrections are often necessary to ensure that margins of safety computed at time of design using nominal load and resistance values are better estimates of the actual margins of safety for the soil-structure limit state under operational conditions. This correction is often not required in structural engineering limit state designs

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Nomenclature

β	reliability index (–)
COV	coefficient of variation (–)
$COV_{\lambda R}$	coefficient of variation of resistance method bias (–)
COV_{R_n}	coefficient of variation of nominal resistance (–)
$COV_{\lambda Qn1}$	coefficient of variation of load 1 method bias (–)
$COV_{\lambda Qn2}$	coefficient of variation of load 2 method bias (–)
$COV_{Q_{n1}}$	coefficient of variation of nominal load 1 (–)
$COV_{Q_{n2}}$	coefficient of variation of nominal load 2 (–)
COV_{Q_m}	coefficient of variation of sum of measured loads (–)
Φ	standard normal cumulative distribution function (–)
$\mu_{\lambda R}$	mean of bias resistance (–)
μ_{R_n}	mean of nominal resistance (R_n)
$\mu_{\lambda Qn1}$	mean of load 1 bias (–)
$\mu_{\lambda Qn2}$	mean of load 2 bias (–)
$\mu_{Q_{n1}}$,μ_{Q_n}	mean of nominal load 1 (Q_{n1})
$\mu_{Q_{n2}}$	mean of nominal load 2 (Q_{n2})
μ_{Q_m}	mean of measured load
P_f	probability of failure (–)
R_m	measured resistance
R_n	nominal resistance
Q_m	measured load
Q_{n1}	nominal load 1
Q_{n2}	nominal load 2
ρ	cross-correlation coefficient (–)
ρ_R	cross-correlation coefficient between nominal resistance and method bias (–)
$\rho_{Q_{n1}}$	cross-correlation coefficient between nominal and method bias of load 1 (–)
$\rho_{Q_{n2}}$	cross-correlation coefficient between nominal and method bias of load 2 (–)
$\sigma_{\lambda Qn1}$	standard deviation of load 1 bias
$\sigma_{\lambda Qn2}$	standard deviation of load 2 bias
$\sigma_{Q_{n1}}$	standard deviation of nominal load 1
$\sigma_{Q_{n2}}$	standard deviation of nominal load 2

but is often necessary using geotechnical soil-structure limit state performance functions which have larger sources of uncertainty.

The paper includes sensitivity analyses that demonstrate the quantitative influence of limit state function accuracy (mean and spread of bias values), dependencies (cross-correlations between bias values and predicted (nominal) load and resistance values), cross-correlations between nominal loads and resistance, and average margin of safety (defined as the operational factor of safety) on computed reliability. The accuracy of the closed-form solution is compared to results using the Monte Carlo method which is more general.

The uncertainty in nominal load and resistance values at time of design is linked to quantities that capture the level of understanding (confidence) associated with the choice of nominal load and resistance values based on load and resistance factor design (LRFD) practice for geotechnical foundations in Canada [10,11].

An advantage of the closed-form solution for the conditions described in this paper, is that the influence of uncertainty in nominal values, bias statistics and cross-correlations on the magnitude of reliability index, is transparent and easily explored using a spreadsheet.

3. Formulation of limit state functions with one resistance term and two load terms

Many limit state functions in geotechnical soil and soil-structure problems can be expressed by simple equations of the form

$$g = R_m - Q_m \tag{1}$$

Here g is a random variable representing the margin of safety, and R_m and Q_m are random uncorrelated measured (actual) resistance and sum of two measured load contributions, respectively. Measured values are used in this expression because the objective of this paper is to express the margin of safety as an estimate of the actual (or true) probability of failure, or equivalently, the reliability index (β). The equivalent performance function with g redefined as g/Q_m is

$$g = R_m/Q_m - 1 \tag{2}$$

where R_m/Q_m is the true (actual) factor of safety. Measured resistance and load values are related to nominal resistance value (R_n) and nominal load values (Q_{n1} and Q_{n2}) through resistance bias (λ_R) and load bias values ($\lambda_{Q_{n1}}$ and $\lambda_{Q_{n2}}$), respectively, as follows

$$R_m = \lambda_R R_n \tag{3a}$$

$$Q_m = \lambda_{Q_{n1}} Q_{n1} + \lambda_{Q_{n2}} Q_{n2} \tag{3b}$$

Substituting Eqs. (3a) and (3b) into Eq. (2) gives

$$g = \left(\frac{\lambda_R R_n}{\lambda_{Q_{n1}} Q_{n1} + \lambda_{Q_{n2}} Q_{n2}}\right) - 1 \tag{4}$$

Assuming all variables on the right side of Eq. (4) are random distributed, then the probability of failure [$P_f = P(g < 0)$] may be computed using Monte Carlo simulation provided statistical quantities describing the mean and spread of each distribution are known. The probability of failure using Monte Carlo simulation is simply the number of times $g < 0$ in a large number of trials.

Soil-structure interaction limit states in geotechnical engineering with no method bias are an unlikely occurrence, particularly for soil-structure interaction problems identified earlier. The situation is further complicated when bias values are not available which is the typical case. Fortunately, there are some soil-structure interaction problems for which bias values are available. For example, dead load bias values from instrumented reinforcement layers in mechanically stabilized earth (MSE) walls due to permanent soil self-weight have been reported by Allen and Bathurst [12] and for soil nails by Lin et al. [13]. Resistance-side bias values for compression piles have been reported by Paikowsky [8] and Allen [2,14]; for the ultimate pullout limit state in MSE walls by Huang and Bathurst [15], Huang et al. [16], Yu and Bathurst [17] and Miyata et al. [30]; and for soil nail pullout by Lin et al. [18].

4. Reliability index β

The probability of failure [$P_f = P(g < 0)$] for performance function g (Eq. (2)) is

$$P(g < 0) = P(R_m/Q_m < 1) = 1 - \Phi(\beta) \tag{5}$$

Here, β is reliability index and Φ is the standard normal cumulative distribution function (NORMSDIST in Excel). Assuming lognormal distributed random variables then

$$\beta = \left(\frac{\mu_{\ln(R_m/Q_m)}}{\sigma_{\ln(R_m/Q_m)}}\right) \tag{6}$$

The numerator and denominator are the mean and standard deviation of the lognormal of the variable computed as (R_m/Q_m), respectively. Derivations presented in the Supplementary Material to this paper lead to a general expression for β which is the reference case 1 in this study as shown below. This equation falls into the category of first-order second-moment type (FOSM) in the reliability-based design literature (e.g. [9]).

Case 1. The general expression for reliability index β is