Novel discrete element modeling coupled with finite element method for investigating ballasted railway track dynamics

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ABSTRACT
A quadruple discrete element method (QDEM) was developed for viscoelastic multi-body dynamics. The sleeper motion modeled using QDEM was coupled with the rail motion modeled using a finite element method (FEM). The traffic impact response of a ballast particle and a sleeper was analyzed. The three-dimensional spatial distribution of ballast particle displacement was clearly revealed. Moreover, the ballast layer absorbed the low frequency vibration of the sleeper more effectively than its high frequency vibration. This study suggests that the proposed QDEM-FEM can provide greater insight into the impact response of ballasted railway tracks.

1. Introduction
A ballasted railway track consists of compacted ballast rocks on a subgrade. The ballast layer fulfills the important role of absorbing and distributing the traffic impact load through sleepers induced by the passing of a train. These important functions are performed by the multi-body contact mechanics of the tightly compacted discontinuous aggregates that propagate and attenuate an elastic wave by external friction or internal viscous damping. However, the impact wave propagations induce complex vibration modes in the ballast layer, and invisible gaps accumulate among the ballast rocks because of the cyclic passage of trains, which has been thought to deteriorate the ballasted track via the loosening and subsidence of the ballast layer. Therefore, ballasted tracks require periodic maintenance, which is an important concern for reducing the maintenance costs associated with railway management.

Many attempts were recently made to improve the ballasted track structure. The ballast layer was reinforced with geosynthetic materials to reduce the railway track settlement \cite{1,2}. In addition, many elastic elements, such as rail pads, under-sleeper pads, and under-ballast mats, were developed to reduce track deterioration \cite{3,4}. In one experimental investigation, the sleeper’s vibration characteristics, including the dynamic effects of sleeper-ballast interactions, were investigated through a modal analysis to predict the railway track’s dynamic response \cite{5}. Dynamic wheel/rail interactions, which significantly contribute to impact vibration and noise, were also investigated for rail and wheel surface defects in field measurements and numerical simulations \cite{6,7}.

In addition, numerical simulations were conducted using a finite element method (FEM) or a discrete element method (DEM) \cite{8}. FEM simulations of continuum mechanics significantly contributed to the vibration analysis of railway tracks under traffic impact loads \cite{9–12}. Simplification techniques that use spring-dashpot elements and/or Timoshenko beam elements enabled important reductions in computational costs \cite{13–16}. In these simulations, the ballast layer is considered as a continuous body mechanics. Therefore, transitions from continuous to discontinuous behavior are difficult to model. Such transitions include disconnections between contacting ballast rocks or ballast rock fractures in the ballast layer, which are considerable causes of track subsidence. Meanwhile, DEMs were adopted to address such discontinuous problems. Ballasted track components were modeled by a bonding particle method to analyze the dynamic characteristics of the ballasted track and simulate the deformation and fractures modeled as the stretching and disjointing of springs between bonding particles \cite{17,18}. The accuracy of the ballast particle shape models was improved \cite{19}, and a simplified model of the ballast particle shape was proposed to simulate a large number of load cycles \cite{20}. Several studies simulated interactions, such as the interlocking of and contact between actual ballast particles, because of their irregular shape, thereby modeling the realistic particle shapes as rigid polyhedrons \cite{21–24}.

In these DEM approaches, the vibration characteristics and absorption of each track component were never discussed even if the effects of cyclic loading on ballast settlement were investigated because DEMs are fundamentally challenged in terms of their capacity to accurately reproduce elastic structural deformations associated with...
internal viscous damping. As a result, the dynamics of ballast layers as discontinuous aggregates were not sufficiently elucidated because the simulations have thus far never focused on the vibrations of individual ballast particles. Nonetheless, these vibration characteristics should be clarified because the elastic and rigid vibration modes of individual ballast particles in contact, which are induced by the traffic impact loads, consequently trigger the deterioration of the ballasted track. Therefore, an advanced method of simulating the contact dynamics of individual ballast particles, including their viscoelastic deformation, must be developed to better understand the causes of the ballasted track deterioration.

In conventional DEM applications, including discontinuous deformation analyses (DDA) [25,26], rigid body spring model (RBSM) [27], and UDEC®/3DEC® [28], the artificial joints connected by additional elastic springs are applied between the blocks or particles to address the deformation or fracture behavior [29–32]. Deformation and fracture are accordingly explained by the stretching and disjointing of the springs. Nevertheless, reproducing the real mechanical properties of a continuum material using only the spring constant between two particles is nearly impossible because the result obtained by the bonding particle model depends not only on the spring constant, but also on the structural properties of the particle assembly, such as the void ratio, coordination number, and particle size distribution. The constitutive laws of the particle assembly were reported for limited cases only [33]. Therefore, the spring constant must usually be adjusted by trial and error to reproduce the actual phenomena. Alternatively, approaches that were developed to model the failure of composite materials, such as a concrete, can be used to improve the modeling accuracy. Such methods include lattice-based mesoscale methods that use a lattice discrete particle model (LDPM) [34,35] or a rigid body spring network (RBSN) [36] approach that improves the RBSM.

In the early 1990s, Munjiza overcame the abovementioned problems by developing the combined finite discrete element method (FDEM) [37]. The FDEM is an effective solution for multi-body dynamics involving viscoelastic deformation and material fracture. This method is widely applied in the fields of geoscience and ge-engineering [38,39]. In addition, Sakaguchi developed the quadruple discrete element method (QDEM) in the early 2000s [40] to simulate Earth’s interior dynamics based on multi-material and multi-rheological properties. Subsequently, we extended the QDEM features to multi-body simulations using a contact algorithm between objects in a DEM [41]. These methods combine the advantages of the continuum model and DEM. In accounting for the computational cost, the QDEM posits a constitutive law of a four-particle element (i.e., quadruple discrete element) as the minimum number of particles required to precisely express the changes in surface area and volume. The viscoelastic stress is explicitly calculated in an element-by-element manner based on a constitutive law [42] that employs the real viscoelastic parameters of the materials. The particles are then individually moved according to their interactions with the surrounding elements.

Although the QDEM algorithm cannot address the fracture process in the manner of the FDEM, the QDEM consumes minimal memory and exhibits high computational efficiency because it does not require the calculation of global matrices as in the case of FEM. In addition, the QDEM algorithm can be made suitable for large-scale parallel computing by adopting a recently presented dynamic load balancing technique [43,44]. QDEM can easily implement high-efficiency parallelization because the method not only calculates the stress in an element-by-element manner, but also calculates individual particle motions using only local components of elements surrounding the particle. The QDEM algorithm is particularly compatible with shared memory parallelization techniques for particle simulation [45,46]. The QDEM algorithm can effectively leverage various current hardware with an increasing number of arithmetic processor cores and threads, such as a graphics processing unit (GPU). Although we also implemented multi-GPU computing in this study, the details of this implementation are omitted herein because they are irrelevant to our present objective.

First, in this study, we analyze the vibration characteristics of the ballast particle and the sleeper under an artificial impulsive loading condition to confirm the reliability of the QDEM simulations of the ballasted railway tracks. We then compare these characteristics with actual in situ measurements taken with a special sensing ballast rock and sensing sleeper [10]. Through these verification tests, we also clarify the significance of the QDEM compared to the ordinary DEM. After the verification, the QDEM simulation of the ballasted track is coupled with a rolling wheel/rail contact FEM simulation to apply the impact load caused by the rail/wheel contact mechanics to the QDEM simulation. Furthermore, we clarify the effects of the vibration characteristics of the ballast particle and the sleeper on their displacement through the QDEM-FEM impact response analysis of the ballasted railway tracks. This study demonstrates that our proposed simulation offers a promising technique for obtaining useful information to produce optimum designs of ballasted railway track structures and avoid deterioration.

2. Discrete element modeling of viscoelastic materials

The interactive forces between two particles in ordinary discrete element models are considered regardless of the dimension in space. For example, in the ordinary DEM, the viscoelastic force is determined as a function of the relative displacement and the velocity of two particles in contact. In the simplest case, such as that of a linear viscoelastic model, the spring constant and the damping coefficient of the Voigt model characterize the interactive force. The bulk behavior shows a perfectly one-dimensional linear viscoelasticity characterized by the spring constant and the damping coefficient when many particles in the neighborhood are bonded according to the Voigt model and form a one-dimensional bar. In contrast, as mentioned in Section 1, the bulk behavior does not match the real three-dimensional (3D) linear viscoelasticity if the bonded particles form a tetrahedron (Fig. 1). Therefore, a four-particle interaction model instead of a two-particle interaction model is introduced as an extension of the discrete element model to overcome this problem. This simple concept is the basis of the QDEM.

We must determine the type of space discretization that should be adopted in QDEM in terms of the original problem of describing the mechanical behavior of continuum materials by discrete element modeling. Under the QDEM concept, a tetrahedron is not a representation of one finite element as in FEM, but is instead a representation of the interacting component among four particles. In this

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Fig. 1. Tetrahedral interaction model following the bonding particle method in the ordinary DEM.