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Research Paper

Discrete modelling jointed rock slopes using mathematical programming methods

J. Meng, J. Huang*, S.W. Sloan, D. Sheng

ARC Centre of Excellence for Geotechnical Science and Engineering, Faculty of Engineering and Built Environment, The University of Newcastle, Callaghan, NSW 2308, Australia

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ABSTRACT

In this paper, a mathematical programming based discontinuum approach is developed for modelling jointed rock slopes. The formulation naturally leads to a standard second-order cone program, which can be solved using efficient optimisation solvers, and a purely static method is derived that does not require artificial damping parameters. Notably, the approach provides somewhat of a unification of two distinct discontinuum approaches: the soft-particle model and the hard-particle model. A series of numerical examples are conducted to validate the proposed approach. The soft-particle model is more versatile than the hard-particle model while the hard-particle model is more efficient.

1. Introduction

The stability analysis of jointed rock slopes is challenging, mainly because of the existence of the discontinuities introduced by joints or faults. Because of its importance, the problem has been widely studied and a large number of models have been proposed. Although limit equilibrium methods [1–3] have traditionally been employed, and are effective in slope stability analysis, they require *a priori* assumptions regarding the force distributions as well as the shape and location of the failure surface. As a consequence, little information about the actual failure mechanism can be obtained. To better simulate the failure mechanism and overall deformation pattern, various numerical models have been proposed in recent decades, including the finite element method (FEM) [4], the boundary element method (BEM) [5], the discrete element method (DEM) [6] and discontinuous deformation analysis (DDA) [7]. The FEM has been widely employed for slope stability analysis (e.g., [8,9] and [10]) with good results, but to model pre-existing discontinuities it is generally necessary to use joint (or interface) elements. Since these elements can lead to difficulties involving remeshing and convergence issues [11], the extended finite element method (XFEM) [12] has subsequently been proposed. This procedure has been applied to model pre-existing discontinuities and crack propagation in jointed rock masses with some success [13,14].

To model discontinuous materials under large deformation efficiently, discontinuum approaches may be used. There are essentially two types of these approaches: the soft-particle model and the hard-particle model [15–17]. The soft-particle model (e.g., the conventional DEM originally developed by Cundall and Strack [6]) considers overlap at the contact points between particles to simulate particle elasticity. This approach is now used widely in the rock mechanics community. Since pre-existing discontinuities and the growth of cracks can be represented explicitly and conveniently, the interaction of these phenomena can be modelled in a straightforward manner [18–20]. The influence of joints' geometrical and mechanical behaviours on jointed rock slopes' stability are investigated as well [21,22]. Furthermore, modelling the complete failure process of a slope is now possible [23,24].

The second discontinuum approach is the hard-particle model e.g., contact dynamics [25,26]. In contrast to the soft-particle model, the overlap is not allowed at contacts (i.e., particles are perfectly rigid) and non-smooth contact laws are used. Furthermore, damping coefficients or local elastic parameters are not necessarily required in the model [16] and large time steps can be employed because of the adoption of implicit time discretisation [26]. Numerous studies have been performed using this model and it has proven to be very successful in modelling granular materials (e.g., [27–31]). Although this discontinuum approach has the potential for modelling jointed rock masses, it has rarely been used for this purpose. One study was performed by Rafiie et al. [32], but all results were limited to Fig. 3 in this reference.

This work further develops the discontinuum approach based on mathematical programming [33–39] for modelling jointed rock slopes. A series of variational formulations are developed for the statics and dynamics of jointed rock slopes. The frictional contact problem for rock

* Corresponding author.
E-mail address: jinsong.huang@newcastle.edu.au (J. Huang).

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Nomenclature

$\boldsymbol{a}_0^s$ acceleration of the seismic loads
A^I and $\boldsymbol{A}$ area of the interface I and array containing all interfaces' area, respectively
c micro-parameter, the cohesion at an interface
$\boldsymbol{C}_n$ and $\boldsymbol{C}_t$ diag$(1/k_n^1,...,1/k_n^N)$ and diag$(1/k_t^1,...,1/k_t^N)$, respectively
$\overline{\boldsymbol{C}}_n$, $\overline{\boldsymbol{C}}_t$ and $\overline{\boldsymbol{C}}_\varphi$ diag$(1/\overline{k}_n^1,...,1/\overline{k}_n^N)$,diag$(1/\overline{k}_t^1,...,1/\overline{k}_t^N)$ and diag$(1/\overline{k}_\varphi^1,...,1/\overline{k}_\varphi^N)$, respectively
D sliding distance of the block relative to the base
$\boldsymbol{d}_0^s$ and $\boldsymbol{d}^s$ displacement constraints of the seismic loads
E and $\hat{E}$ elastic modulus and associated micro-parameter
$\boldsymbol{f}_{ext}^i$ and $\boldsymbol{f}_{ext}$ external force vector of the ith block and its corresponding matrix for all blocks
f_n^I and $\boldsymbol{f}_n$ tensile force limit of interface I and the corresponding matrix for all interfaces
g, g_0 and $\boldsymbol{g}_0$ contact gap at the current and next time step and the matrix containing all the interface gaps
J^i, $\overline{J}^i$ and $\overline{\boldsymbol{J}}$ mass moment of inertia the ith block, $\frac{J^i}{\theta\Delta t^2}$, and the corresponding matrix for all blocks
k_n and k_t normal and tangential contact stiffness for frictional contacts
$\overline{k}_n$, $\overline{k}_t$ and $\overline{k}_\varphi$ normal, tangential and rotational spring stiffnesses for bonded blocks
l_n and l_t normal and tangential length of an interface
$l_{\min}$ the minimum distance between any two random points
m_{ext}^i external moment for the ith block
m^i, $\overline{\mathrm{m}}^i$ and $\overline{\boldsymbol{M}}$ mass of the ith block, $\frac{\mathrm{m}^i}{\theta\Delta t^2}$, and its corresponding matrix for all blocks
$\boldsymbol{n}_0^I$ and $\hat{\boldsymbol{n}}_0^I$ unit normal and tangential vectors at interface I
$\boldsymbol{N}_0$ and $\hat{\boldsymbol{N}}_0$ matrixes containing unit normal and tangential vectors for all interfaces
$\overline{\boldsymbol{N}}_0$ linear mapping matrix of rotational torques from local contact variables to the global-coordinate system
O, O_i and O_j midpoint of the interface for the initial configuration, block i and block j
p^I and $\boldsymbol{p}$ normal contact force at contact I and the array containing the normal reaction forces at all interfaces
q^I and $\boldsymbol{q}$ tangential contact force at contact I and the array containing the tangential reaction forces at all interfaces
$\overline{p}^I$ and $\overline{q}^I$ reaction forces at interface I in the normal and tangential directions
$\overline{\boldsymbol{p}}$ and $\overline{\boldsymbol{q}}$ matrices collecting all reaction forces in the normal and tangential directions
$\boldsymbol{r}$ dynamic force i.e., $\overline{\boldsymbol{J}}\Delta\boldsymbol{\alpha}$
R_{ip}^I and R_{iq}^I moment arms of the reaction forces p^I and q^I for the block i at contact I
$\boldsymbol{R}^{\mathrm{p}}$ and $\boldsymbol{R}^{\mathrm{q}}$ matrixes containing all moment arms i.e., R_{ip}^I and R_{iq}^I
$\boldsymbol{R}^{\overline{\mathrm{p}}}$ and $\boldsymbol{R}^{\overline{\mathrm{q}}}$ matrices containing moment arms for $\overline{\boldsymbol{p}}$ and $\overline{\boldsymbol{q}}$, respectively.
$\boldsymbol{t}$ dynamic force i.e., $\overline{\boldsymbol{M}}\Delta\boldsymbol{x}$
t time
$\boldsymbol{v}_0^i$ and $\boldsymbol{v}^i$ current and next velocity of the ith block
$\boldsymbol{v}_0^s$ and $\boldsymbol{v}^s$ velocity of the seismic loads
V sliding velocity of the block relative to the base
w_0^i and w^i current and next angular velocity of the ith block
$\boldsymbol{x}_0^i$, $\boldsymbol{x}^i$ and $\boldsymbol{x}$ current and next position of the ith block and the matrix containing all the block positions
α_0^i, α^i and $\boldsymbol{\alpha}$ current and next angular position of the ith block and the matrix containing all the block angular positions
Δt time step
Δu_n, Δu_t and $\Delta\alpha$ local relative displacements at interface I in the normal, tangential and rotational directions.
$\Delta\boldsymbol{u}_n$, $\Delta\boldsymbol{u}_t$ and $\overline{\boldsymbol{N}}_0^{\mathrm{T}}\Delta\boldsymbol{\alpha}$ local relative displacements at interfaces in the normal, tangential and rotational directions.
$\Delta\boldsymbol{x}$ and $\Delta\boldsymbol{\alpha}$ linear and angular displacements of blocks at global level
$\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ arrays containing Lagrange multipliers
μ friction coefficient of the frictional blocks
μ_b micro-parameter, friction coefficient of the bonded blocks, $\mu_b = \tan\phi_b$
ν and $\hat{\nu}$ Poisson's ratio and the associated micro-parameter
σ_t micro-parameter, the tensile strength of an interface
$\overline{\tau}^I$ and $\boldsymbol{\tau}$ torque at the interface I and the array containing torques at all interfaces
ϕ_b and ϕ_c local friction angle of intact interfaces and failure interfaces, respectively

blocks is first considered. To model failure of intact rock materials, a polygonal discretisation is used which is based on the Voronoi diagram. The variational formulation is developed based on the rigid-body-spring network [40]. All the formulations developed can be cast into standard second-order cone programs, which can be solved conveniently using advanced optimisation algorithms. A number of very efficient and robust second-order cone programming codes have been developed recently, including MOSEK [41] and SeDuMi [42], which are ideally suited to the proposed formulations. It is notable that this approach is more general than the conventional DEM because it incorporates both the hard-particle and the soft-particle models. Thus, these two models can be implemented for jointed rock slopes and their results are compared. Another distinct feature is that the purely static formulations, dedicated to modelling quasi-static problems, can be formulated.

This paper is organised as follows. In Section 2, formulations for frictionless blocks with the hard-particle model are developed and then extended to the frictional case. These formulations are further extended to the soft-particle model in Section 3. To model intact rock materials, the governing equations for the bonded block model with the modified Mohr-Coulomb failure criterion are summarised in Section 4. Implementation details are presented in Section 5. In Section 6, the proposed approach is validated with four examples by comparing the numerical results with analytical solutions and experimental observations. Results from the soft-particle model and the hard-particle model are also compared. Moreover, the capability of the proposed approach with the soft-particle model is tested with a jointed rock slope containing two sets of non-persistent joints. Finally, some conclusions are drawn in Section 7.

2. Frictional blocks with the hard-particle model

The formulations developed in this section are aimed at describing sticking, sliding and separating of rock blocks. For clarity, the variational formulations for frictionless blocks are developed first and only the translational degrees-of-freedom are considered. Then, the governing equations are extended to handle frictional blocks by accounting for tangential forces and the rotational degrees-of-freedom.

2.1. Frictionless blocks with the hard-particle model

2.1.1. Equations of motion

The equations of motion for a single block i are given by:

$$\mathrm{m}^i\dot{\boldsymbol{v}}^i = \boldsymbol{f}_{ext}^i \tag{1}$$

where m^i is its mass, $(\boldsymbol{v}^i)^{\mathrm{T}} = [v_x^i, v_y^i]$ is its linear velocity, $\boldsymbol{f}_{ext}^i$ is the external force vector acting on it.