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Research Paper

Robust estimation of the fracture diameter distribution from the true trace length distribution in the Poisson-disc discrete fracture network model

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ABSTRACT

A distribution-free approach is proposed to estimate the fracture size distribution from a given trace length distribution, assuming that the fracture network can be represented by a Poisson-disc model. This approach directly works on the experimental distribution of the observed trace lengths (corrected from sampling biases) and does not need to choose a parametric model for the trace length or fracture diameter distributions. It is more robust than existing models and provides an unbiased estimate of the cumulative distribution function of the fracture diameters. Its simplicity of use, accuracy and versatility are illustrated through synthetic examples.

1. Introduction

A comprehensive understanding of fracture networks is critical to the economic development of underground mining (cave ability, water drainage, roof stability, fragmentation, gas ventilation, flow gravity), open pit mining (slope stability, water drainage, blast ability, solution mining, in situ leaching), tailing dam (environmental aspects), oil and gas reservoir engineering (fractured reservoirs and unconventional reservoirs), generation of heat and vapor from geothermal reservoirs, management of groundwater resources and underground nuclear wastes disposal. The characterization of fracture networks is one of the most important parts of the engineering characterization of rock masses. The fracture properties that have the greatest influence at the design stage are location, orientation, size, frequency, surface geometry, genetic type and infill material [31,32].

One of the most interesting parameters of fracture networks is the fracture intensity, i.e. the mean area of fractures per unit volume [10]. A fixed fracture intensity can be the result of very different scenarios, such as a network of many small fractures or a network of few large fractures. As an example, for a given fracture intensity, there is a better connectivity with a few large fractures than with many small ones and, in terms of fragmentation, the size of blocks that are results of the intersections of the fracture network will change with different scenarios of the fracture sizes. Hence it is important to accurately estimate the distribution of fracture sizes.

To reach this objective, the available information usually consists of surface observations, namely trace lengths measured on exposures, such as natural outcrops, rock cuts and tunnel walls. One can distinguish three types of surface samplings: (1) scan line sampling that measures the trace lengths of the fractures that intersect a line drawn on the exposure; (2) circle sampling that measures the trace lengths of the fractures that intersect a circle drawn on the exposure; and (3) window or area sampling that measures the trace lengths of the fractures within a finite size area (such as a rectangle or a circular window) [3,11,8].

This paper focuses on the estimation of the fracture size distribution from the trace length distribution. It is outlined as follows. Section 2 presents the hypotheses and problem setting and a review of existing approaches for modeling the fracture size distribution knowing the trace length distribution. An alternative approach and its computational implementation are then introduced in Section 3. Numerical experiments on simulated fracture networks are presented in Section 4 to demonstrate the applicability, accuracy and robustness of the proposed approach. A general discussion and conclusions follow in Section 5.

2. State of the art

2.1. Modeling assumptions and problem statement

By setting up an object at each point of a 3D Poisson process, a Boolean model is obtained [27,28]. This object can be the same at every point or can be random, with a different shape, size and/or orientation. For fracture networks, the Boolean model often uses a circular disc as the object, which is known as the Poisson-disc model. This model has been first used for rock mechanics application by Baecher et al. [2]. The model parameters are the intensity of the Poisson point process, which

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gives the average density of the points located in some region of space, and the joint distribution of the fracture orientations and fracture diameters. To enrich the model, other parameters such as fracture aperture and thickness could also be considered, which can be correlated with the fracture diameters [26,6], but will be out of the scope of this work.

The Poisson-disc model relies on the following assumptions:

(1) The fractures are modeled as two-dimensional circular discs scattered in the three-dimensional space, with random positions, diameters and orientations.
(2) The fracture diameters are independent and identically distributed (i.i.d.).
(3) The fracture orientations are independent and identically distributed. We do not assume any specific restriction on this distribution; in particular, the distributions of the fracture dips and dip directions can cover the full ranges of angles (0-90° for the dips and 0-360° for the dip directions) or any parts of these ranges.
(4) The fracture centers form a Poisson point process whose intensity is constant in space (homogeneous process).

These assumptions are basically the same as that used by Baecher et al. [2], Kulatilake and Wu [21], Zhang and Einstein [45], Song and Lee [40], Jimenez-Rodriguez and Sitar [18] and Song [38], among others. Using stereological considerations, Warburton [44] established the following relationship between the diameter distribution of the fractures with given orientation α (direction of the fracture pole, represented as a point of the 2-sphere S_2) and the distribution of their trace lengths on a given plane P:

$$f_\alpha^{(P)}(l) = \frac{l}{\mu_D(\alpha)} \int_l^{+\infty} \frac{g_\alpha(\delta)d\delta}{\sqrt{\delta^2 - l^2}} \tag{1}$$

where $f_\alpha^{(P)}(l)$ is the probability density function (for short, *pdf*) of the trace lengths on plane P induced by the fractures with orientation α, while $\mu_D(\alpha)$ and $g_\alpha(\delta)$ are the expected value and the *pdf* of the diameters of such fractures with orientation α.

If one further assumes that the fracture diameter and fracture orientation are independent, then $g_\alpha(\delta)$ is actually independent of α and can be denoted as $g(\delta)$. Under this additional assumption, the *pdf* of the trace lengths induced by all the fractures on plane P is found by integrating Eq. (1) over all the possible orientations on the 2-sphere S_2, which gives

$$f(l) = \frac{l}{\mu_D} \int_l^{+\infty} \frac{g(\delta)d\delta}{\sqrt{\delta^2 - l^2}} \tag{2}$$

where $f(l)$ is the *pdf* of the trace lengths on plane P induced by all the fractures, irrespective of their orientations, μ_D is the mean fracture diameter, and $g(\delta)$ is the *pdf* of the fracture diameters.

Note that the probability density function f does not depend on the particular plane P that has been chosen for observing the fracture trace lengths, as long as fracture diameters and fracture orientations are independent [42]. It corresponds to the *pdf* of the trace lengths observed on the whole plane P (equivalently, on a sampling window with an infinite size and with fixed and known orientation), which will be referred to as the "true" trace length *pdf*.

In practice however, trace lengths are measures in sampling windows with finite sizes and only a fraction of the fracture traces within the window may be measured. Accordingly, the measured trace length distribution usually differs from the true trace length distribution and suffers from orientation, size, truncation and censoring biases [3,11,33,21]. Several approaches have been proposed in the past decades to correct these sampling biases, which are out of the scope of this work (e.g., [44,33,30,25,21,43,29,34,40,46]).

In the following, the true trace length distribution (after correcting from sampling biases) will be supposed known. The problem is therefore to invert the integral in Eq. (2), in order to express the diameter probability density function (g) as a function of the true trace length probability density function (f).

Before proceeding, it is interesting to mention that the previous model assumptions can be weakened. First, the results further presented remain valid if the fracture diameters, fracture orientations and Poisson intensity are replaced by independent stationary ergodic random fields. Ergodicity guarantees that the experimental distributions observed over a large sampling domain are representative, up to statistical fluctuations, of the underlying model distributions, similarly to what happens with i.i.d. random variables [6,5]. The only requirement is thus to assume that the sampling window is large enough to observe the full distribution ranges of diameters, orientations and intensity. Note that randomizing the Poisson intensity converts the homogeneous Poisson process representing the fracture centers into a doubly stochastic Poisson process, also known as a Cox process.

Second, if the fracture diameters do not have an absolutely continuous distribution (e.g., if only a discrete set of fracture diameters are possible), one has to replace the numerator of the integrand in Eq. (2) ($g(\delta)d\delta$) by $dG(\delta)$, where $G(\delta)$ stands for the cumulative distribution function (*cdf*) of the fracture diameters. Up to this formal modification, all the equations and demonstrations presented hereafter remain valid. Note that, under the abovementioned assumptions for the Poisson-disc model, the trace length distribution is absolutely continuous and possesses a probability density function f that takes finite values except for the discontinuity points of G, as per Eq. (2), even if the density $g(\delta)$ takes infinite values for some specific values of δ or it is undefined (case of a non-absolutely continuous diameter distribution).

Third, if the fracture diameter and fracture orientation are not independent, Eq. (2) is no longer valid and one has to use Eq. (1) instead. One can therefore determine the diameter distribution of the fractures with a given orientation α (density g_α) by considering only the trace lengths on the sampling plane P induced by the fractures with orientation α (density $f_\alpha^{(P)}$), and repeat the procedure for different choices of the fracture orientation on the 2-sphere S_2. In practice, it is usual to distinguish a few fracture sets comprising approximately parallel fractures of the same type and age (for instance, fracture clusters whose distribution of orientations has a small dispersion). Based on Eq. (1), the formalism hereafter described can be applied to each fracture set separately.

2.2. Inverse and forward modeling

An inverse relationship for Eq. (2) is well-known in the fields of stereology and stochastic geometry [36,37,19,16]:

$$g(\delta) = -\frac{2\mu_D\delta}{\pi} \int_\delta^{+\infty} \frac{1}{\sqrt{l^2 - \delta^2}} \frac{d}{dl}\left\{\frac{f(l)}{l}\right\} dl \tag{3}$$

This equation has been used by Tonon and Chen [41] to obtain the explicit expressions of the fracture diameter distribution for several commonly used trace length distributions (uniform, exponential, gamma and power law). For other trace length distributions such as the lognormal, the same authors propose a numerical approximation of Eq. (3). Other numerical methods have been suggested by Kulatilake and Wu [22], Song and Lee [40], Song [38], Song [39] and Zhu et al. [46], among others, to obtain the probability density function $g(\delta)$ of the fracture diameter distribution using the relationship between trace length and fracture diameter distributions (Eqs. (1) and (2) or equivalent stereological relationships). However, Eq. (3) involves the derivative of $f(l)/l$, the estimation of which lacks robustness in practice: a small variation of $f(l)$ may indeed produce a large variation of the derivative of $f(l)/l$, thus a large variation of $g(\delta)$, making the problem of