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Research Paper

Sequential reduction of slope stability uncertainty based on temporal hydraulic measurements via the ensemble Kalman filter

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ABSTRACT

A data assimilation framework, utilising measurements of pore water pressure to sequentially improve the estimation of soil hydraulic parameters and, in turn, the prediction of slope stability, is proposed. Its effectiveness is demonstrated for an idealised numerical example involving the spatial variability of saturated hydraulic conductivity, . It is shown that the estimation of generally improves with more measurement points. The degree of spatial correlation of influences the improvement in the predicted performance, as does the selection of initial input statistics. However, the results are robust with respect to moderate uncertainty in the spatial and point statistics.

1. Introduction

The slope stability of an embankment subjected to cyclic water level fluctuation is crucial in geotechnical engineering [13,20,22], with the distribution of pore water pressure (PWP) under seepage being particularly relevant in any slope stability assessment [2,28]. To accurately estimate the PWP, a precise determination of the soil hydraulic parameters is required. However, because it is not realistic to conduct in-situ testing everywhere, some uncertainty remains due to the spatial variability of material properties between measurement locations. This causes difficulty in accurately predicting the seepage behaviour and distribution of pore pressures, and, thereby, the embankment stability.

Data assimilation, which can utilise field measurements, is one method of improving the prediction of slope behaviour, because it can improve the estimation of soil parameters. Data assimilation is defined here as any method to include measured data into numerical analyses. Often, a type of data assimilation known as back-analysis is used, where parameters for the analysis are estimated using measured data available at a certain time (normally the end of the period under consideration). Most previous studies related to slope back-analysis have focused on soil shear strength parameters [8,15,25], in which the utilised measurements were mainly displacement or stress/strain. PWP measurements are seldom used in geotechnical engineering, although, in hydrology, it has already been proven that such measurements improve the estimation of hydraulic parameters [27]. In geotechnical engineering, the improved accuracy of hydraulic parameters not only benefits the estimation of PWP but also the prediction of slope stability [24].

A limited number of studies have investigated the influence of improved estimation of hydraulic parameters on slope stability, although they have usually ignored the spatial variability of parameter values. For example, Zhang et al. [26] applied the Bayesian method to back-calculate hydraulic parameters by utilising PWP measurements and investigated the effect of uncertainty in the parameters on the prediction of slope stability, but without incorporating spatial variability. In contrast, Vardon et al. [24] linked the ensemble Kalman filter (EnKF) [4,5] with the random finite element method (RFEM) [9] in steady state seepage to back-calculate the hydraulic conductivity based on PWP measurements. They cross-correlated hydraulic conductivity with the strength parameters (cohesion and friction angle) and investigated the influence of the improved estimation of hydraulic conductivity on the distribution of the factor of safety (FOS). Meanwhile, Jafarpour and Tarrahi [14] indicated that an imprecise knowledge of the spatial continuity could induce erroneous estimations of soil property values, whereas Pasetto et al. [19] investigated the influence of sensor failure on the estimation of k_{sat}, focusing on two cases with different correlation lengths. The results demonstrated that the identification of k_{sat} was more accurate for the larger correlation length. Hommels et al. [12] compared the EnKF with the Bayesian method and concluded that the EnKF, essentially a step-wise Bayesian method, was easier to implement, as it does not require the assimilation of all available data and could sequentially improve the estimation of parameters once further data become available.

In this paper, the authors account for the spatial variability of k_{sat}, which plays a dominant role in rainfall infiltration as pointed out by Rahardjo et al. [21]. In addition, the EnKF is applied to improve the

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Nomenclature

c'	effective cohesion
e	superscript indicating ensemble mean
E	stiffness
FOS	factor of safety
G_s	specific gravity of the soil particles
h_s	suction head
$h_{s,ae}$	air-entry suction head
i	Gauss point number
k_{sat}	saturated hydraulic conductivity
l	scale of fluctuation
l_h	scale of fluctuation in the horizontal direction
l_v	scale of fluctuation in the vertical direction
$\ln k_{sat}$	natural log of k_{sat}
n	fitting parameter of the soil water retention curve
N	total number of ensemble members
N_k	number of unknown k_{sat} values
nn	number of element nodes
PWP	pore water pressure
r	superscript indicating 'real' values
RMSE	root mean square error
SOF	scale of fluctuation
SPREAD	measure of uncertainty between the ensemble members
SWRC	soil water retention curve
t	time
T_1	period of the first sinusoid
T_2	period of the second sinusoid
VAR(i)	ensemble variance for each $\ln k_{sat}$
WL	water level
x	coordinate in the horizontal direction
z	coordinate in the vertical direction
α_d	approximately the inverse of the air-entry suction head for soil water retention curve
θ	volumetric water content
θ_s	saturated volumetric water content
θ_r	residual volumetric water content
μ	mean
σ	standard deviation
υ	Poisson's ratio
ψ	Dilation angle
φ'	effective friction angle

estimation of the k_{sat} field by using (in this instance, numerically generated) 'measurements' of PWP. Due to the existence of spatial variability, the spatial correlation length and arrangement and number of measurement points can have an influence on the data assimilation. Therefore, these aspects are also investigated.

The paper is organised as follows. Firstly, the formulations of stochastic transient seepage, the EnKF and slope stability are introduced. Then, a synthetic example is analysed, to demonstrate the sequential reduction of the uncertainty in k_{sat} and the influence on the subsequent prediction of slope stability. Finally, an investigation into the influence of the pointwise statistics and spatial continuity of k_{sat} on the data assimilation process via the EnKF, utilising synthetic data, has been undertaken.

2. Formulation

2.1. Framework of the overall analysis

Vardon et al. [24] utilised hydraulic measurements in steady-state seepage to reduce slope stability uncertainty via the EnKF. The formulation of the numerical approach was also given. This paper extends the research to transient seepage, as illustrated by the framework shown in Fig. 1.

With reference to Fig. 1(a), the analysis starts by generating an initial ensemble of realisations of the spatial variation of k_{sat}, based on the probability distribution and scales of fluctuation of k_{sat} (i.e. multiple random field realisations of k_{sat} are generated). The initial ensemble of k_{sat} is imported into a stochastic transient seepage process. When the time t reaches t_1, the measurements that have been acquired from the field can be used in the data assimilation process; that is, the EnKF is applied to improve the estimation of k_{sat} for all realisations in the ensemble, based on the measured data. The slope reliability can also be calculated, although, as it is the first time the EnKF is used in the transient seepage process, there is no immediate improvement in the estimated pore pressure. The two options are represented by calculation boxes A and B in Fig. 1(b) and (c), respectively. The analysis then continues until the time reaches t_2, whereupon the computation of pore water pressure resulting from the improved estimation of k_{sat} (calculated at t_1) can be used to compute the slope reliability. At the same time the EnKF can again be applied to get an updated estimation of k_{sat}, since new PWP measurement data have been acquired. As the analysis proceeds still further, the data assimilation continues to t_3, t_4 and so on, with calculation box A or B being followed at each stage.

2.2. Slope stability assessment under transient seepage

The governing equation of 2D transient unsaturated–saturated flow is based on mass conservation, as described in Liu et al. [16,17]. To solve it, both the soil water retention curve (SWRC), which describes the relationship between the suction head, h_s, and the volumetric water content, θ, and the saturated–unsaturated hydraulic conductivity relationship are necessary. In Liu et al. [16,17], the Van Genuchten–-Mualem model [18,23] was used to describe the relationship between h_s and θ, and the impact of hysteresis was examined. Herein, the effect of hysteresis is not taken into account, in order to simplify the computation. The hydraulic conductivity of an unsaturated soil can also be derived using the Van Genuchten [23] model. Fig. 2(a) and (b) shows the volumetric water content and hydraulic conductivity of the unsaturated soil, respectively, as functions of the suction head.

As in Liu et al. [16,17], Bishop's effective stress, incorporating the influence of both suction and water content, has been combined with the extended Mohr–Coulomb failure criterion to calculate the shear strength.

2.3. Soil parameter random fields

The spatial variability of soil parameters is simulated by the generation of random fields, which are based mainly on the statistical distributions and spatial correlations of the parameters. The distribution of a soil parameter is often assumed to be normal or log-normal, and characterised by the mean and standard deviation. In this paper, the distribution of k_{sat} is considered to be log-normal [9,28], so that the natural log of k_{sat}, $\ln k_{sat}$, follows a normal distribution. The spatial correlation of soil parameters is here characterised by the scale of fluctuation (SOF), l, which is the distance over which parameters are significantly correlated, and the exponential correlation function. A more detailed description of the SOF and exponential correlation function are given in Fenton and Griffiths [6].

In this paper, the random fields have been generated using local average subdivision (LAS) [7], using the computer module implemented by Hicks and Samy [10,11]. After the random fields of soil parameters (in this case k_{sat}) have been generated, the values are imported into the finite element program at the Gauss point level and then used in computing the seepage and/or slope stability behaviour. The