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Research Paper

Three-dimensional discontinuous deformation analysis based on strain-rotation decomposition

Huo Fan[a,*], Hong Zheng[b], Jidong Zhao[a,*]

[a] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong
[b] State Key Laboratory of Geomechanics and Geotechnical Engineering, Chinese Academy of Sciences, Wuhan 430071, China

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ABSTRACT

The strain and local rotation at any material point in a deformable body can be descripted by the strain-rotation (S-R) decomposition theorem. This paper presents a three-dimensional dynamic deformation formulation based on the S-R decomposition. The three-dimensional dynamic analysis formulation is generic and can be easily implemented into numerical methods. By combining the new formulation with the discontinuous deformation analysis (DDA), a new method named SR-3D-DDA is developed. We further use several examples to demonstrate that the S-R based DDA can help effectively eliminate the nonphysical volume change exhibited by existing DDAs and improve the accuracy of the predictions.

1. Introduction

Modern engineering design requires numerical tools to be developed in three-dimensional (3D) to be truly predictive. 3D formulations have therefore been implemented in mainstream numerical methods (see e.g. [1–19]). Frequently, numerical predictions of a practical 3D problem need to address challenges pertaining to various nonlinear behaviors, including material nonlinearity, contact nonlinearity and geometric nonlinearity. The conventional popular approaches include the total Lagrangian formulation (T.L.) and the updated Lagrangian formulation (U.L.) are all based on the Green's strain and polar decomposition theorem [20]. Recently, a new dynamic analysis formulation, based on strain-rotation (SR) decomposition theorem, has been proposed [21] to tackle geometric nonlinearity. It has demonstrated an advantage in simultaneously capturing the strain and local rotation reasonably well. However, it is only limited to two-dimensional cases. Meanwhile, as an alternative to describe the dynamic behavior of discontinuous media such as rock that involving discrete block system, 2D discontinuous deformation analysis (2D-DDA) [22] has been developed and has been extended to 3D as well [23]. DDA is able to conveniently simulate the translation, rotation and contact of blocks, while the fundamental unknowns can be made independent of the shapes of blocks. Various techniques have been developed to address the nonlinear behaviors, in particular the contact nonlinearity, based on new contact models [24] and contact resolution or detection algorithms [25–30]. A nodal-based 3D-DDA [31] and a particle-based 3D-DDA were further proposed [32] to enhance the predictive capability of DDA to deal with the deformation of blocks. The bonding and cracking algorithm aiming at 3D particles were implemented [33].The some latest advances in DDA can be found in [34]. However, there have been relatively scarce studies addressing the geometric nonlinearity in DDA.

An apparent pitfall in both 2D [35] and 3D [36] DDA methods is the false volume expansion predictions due to the adoption of first-order rotation approximation. A variety of mitigation methods have been proposed in the past for DDA, including the displacement adjustment method [35], the Taylor series method [37], the trigonometric method [38], and the displacement-strain modification method [39]. Amongst them, the 3D displacement adjustment method [36] appears to perform more robustly in suppressing the unreasonable volume expansion in 3D-DDA. However, in cases of "dual-axial rotation" and "tri-axial rotation" (to be defined in Section 4), the predictions by 3D displacement adjustment method may potentially result in nonphysical expansions in the direction(s) perpendicular to the rotation axis and nonphysical contractions in the direction parallel to the same rotation axis. So the geometrical shape of discrete block is forced to change non-physically though the volume of the block remains the same. The abovementioned expansion and contraction associated with 3D displacement adjustment method seems to have never been mentioned in the literature.

This study presents a substantial extension of the 2D dynamic deformation formulation previously proposed by the authors [21] to three-dimensional case. To effectively address the issue of geometric nonlinearity, a new formulation based on the S-R (strain-rotation) decomposition theorem [40–44] is developed which is generic and readily

* Corresponding authors.
E-mail addresses: huofan_hkust@163.com (H. Fan), jzhao@ust.hk (J. Zhao).

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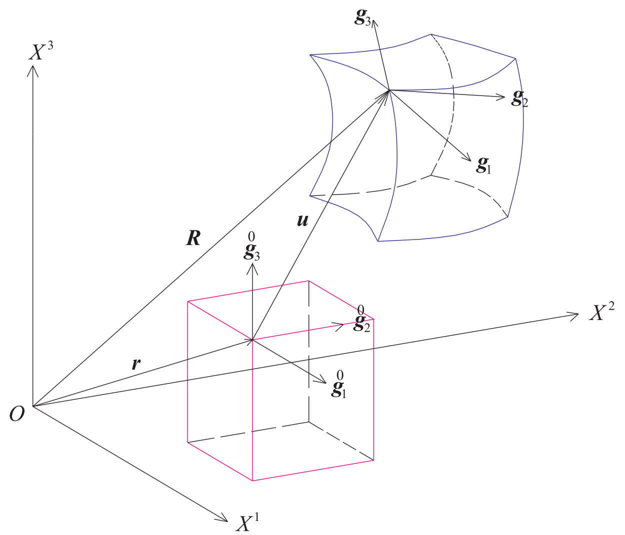

Fig. 1. A deformable body in three-dimension Euclidean space.

applicable to numerical method. The new S-R formulation is then implemented into 3D-DDA to form a new method termed SR-3D-DDA. Several examples are further presented to demonstrate that the advantage of the SR-3D-DDA in capturing the geometric nonlinearity of blocks. The study can offer an effective method in tackling a wide range of engineering problems involving discontinuous materials.

2. Three-dimensional dynamic formulation based on S-R decomposition

Considering the following deformable body in an Euclidean space in Fig. 1, where $\boldsymbol{r}$ and $\boldsymbol{R}$ are the position vectors of a point before and after deformation, respectively; and $\boldsymbol{u}$ is the displacement vector; $\overset{0}{\boldsymbol{g}}_i$ and $\boldsymbol{g}_i$ (i = 1, 2, 3) represent two local basic vectors at a point corresponding to the co-moving coordinate system before and after deformation, respectively.

The S-R decomposition theorem [40–44] states the following decomposition of deformation gradient $\boldsymbol{F}$ into the strain tensor and rotation tensor:

$$\boldsymbol{F} = \boldsymbol{S} + \boldsymbol{R} \tag{1}$$

where the strain tensor is defined by

$$S^i_j = \frac{1}{2}(u^i|_j + u^i|^{\mathrm{T}}_j) - L^i_k L^k_j(1-\cos\theta) \tag{2}$$

and the rotation tensor is

$$R^i_j = \delta^i_j + L^i_j \sin\theta + L^i_k L^k_j(1-\cos\theta) \tag{3}$$

where δ^i_j is the Kronecker-delta. In Eqs. (2) and (3), L^i_j is defined by

$$L^i_j = \frac{1}{2\sin\theta}(u^i|_j - u^i|^{\mathrm{T}}_j) \tag{4}$$

and the average rotation angle θ is determined according to

$$\sin\theta = \frac{1}{2}\sqrt{(u^1|_2 - u^1|^{\mathrm{T}}_2)^2 + (u^2|_3 - u^2|^{\mathrm{T}}_3)^2 + (u^1|_3 - u^1|^{\mathrm{T}}_3)^2} \tag{5}$$

The rotation in a deformed body can be generally described by three methods: (1) the coordinate axis method, i.e., according to [45] (figure 16.1 therein); (2) the quaternion method [46–48]; and (3) the axis-angle method [45,49]. In this study, a rotation matrix or tensor is expressed through a unit rotation axis vector ($\boldsymbol{p}$) and a rotation angle (α) about the axis. For a rigid body rotation, the average rotation angle (θ) in the S-R decomposition is exactly the rotation angle (α) [40]. For the rotation of deformable body, the average rotation angle has a more profound meaning. Several typical examples were given by [40] to

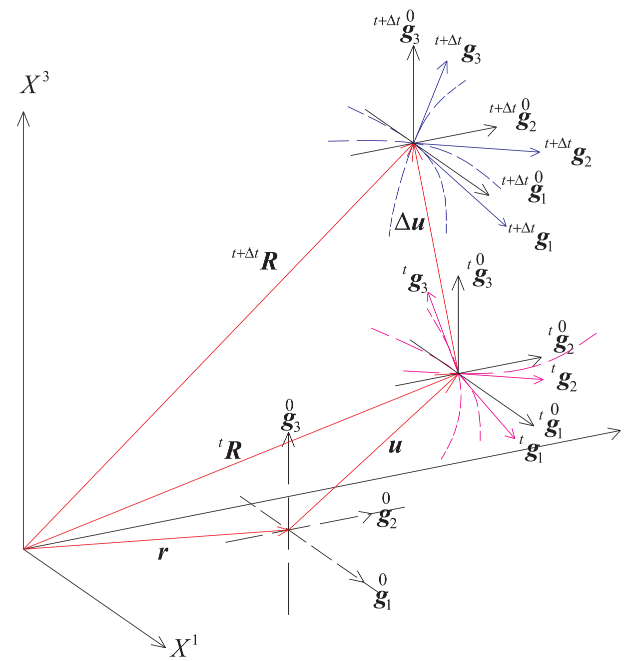

Fig. 2. Update of the three-dimension co-moving coordinate.

illustrate the features of the average rotation angle.

An updated co-moving coordinate as shown in Fig. 2 is adopted, where the superscript " t" and " $t+\Delta t$" correspond to the two consecutive time t and $t+\Delta t$, respectively. In S-R decomposition theorem [43], the symmetric stress is work-conjugate to the strain defined by Eq. (2). Considering a deformable body and applying the principle of virtual displacement, the incremental governing equation can be expressed as [21]

$$\int_{t\Omega} \bar{\sigma}^i_j \delta(\Delta\bar{S}^j_{Li} + \Delta\bar{S}^j_{Ni}) d\Omega + \int_{t\Omega} \bar{D}^{ik}_{jl}(\Delta\bar{S}^l_{Lk} + \Delta\bar{S}^l_{Nk})\delta(\Delta\bar{S}^j_{Li} + \Delta\bar{S}^j_{Ni}) d\Omega + \bar{\bar{F}}_{\mathrm{ine}} + \bar{F}_{\mathrm{pen}} - \bar{F}_{\mathrm{ext}} = 0 \tag{6}$$

where $\bar{\sigma}^i_j$ is the stress, $\Delta\bar{S}^j_{Li}$ and $\Delta\bar{S}^j_{Ni}$ are the linear and nonlinear strain increments, respectively. $\bar{D}^{ik}_{jl}$ is the material tensor associated with the rate-form constitutive laws. Moreover $\bar{\bar{F}}_{\mathrm{ine}}$ and $\bar{F}_{\mathrm{pen}}$ are the virtual work corresponding to the inertia force, constraint force of specified displacement, respectively. $\bar{F}_{\mathrm{ext}}$ represents the external force including the surface and body forces. The hat-lines " $-$" and " $=$" indicate that the variable with respect to basic vectors ${}^t\overset{0}{\boldsymbol{g}}_i$ and ${}^{t+\Delta t}\overset{0}{\boldsymbol{g}}_i$, respectively. For more details one can refer to [21].

The two-dimension problem has been addressed in [21], the three-dimension discretization format will be deduced here. For discretization of space domain, the same interpolation matrix $\boldsymbol{N}(x,y,z)$ can be employed for expressing displacement $\boldsymbol{u}$, velocity $\boldsymbol{V}$ and acceleration $\boldsymbol{A}$. It should be pointed out that the expressions of $\boldsymbol{N}(x,y,z)$ is dependent on the specific numerical method and the mesh topology.

Consider an arbitrary discrete unit, the displacement and displacement increment vectors related to the discrete unit can be denoted by $\boldsymbol{u}$ and $\Delta\boldsymbol{u}$, respectively. Reconsidering Eq. (2), at any point in the discrete unit, the strain vector $\boldsymbol{S}$ can be written as

$$\boldsymbol{S}(x,y,z) = \begin{Bmatrix} S^1_1 \\ S^2_2 \\ S^3_3 \\ 2S^2_3 \\ 2S^3_1 \\ 2S^1_2 \end{Bmatrix} = \boldsymbol{S}_L + \boldsymbol{S}_N = \begin{Bmatrix} u^1|_1 \\ u^2|_2 \\ u^3|_3 \\ u^2|_3 + u^3|_2 \\ u^3|_1 + u^1|_3 \\ u^1|_2 + u^2|_1 \end{Bmatrix} + \begin{Bmatrix} -L^1_k L^k_1(1-\cos\theta) \\ -L^2_k L^k_2(1-\cos\theta) \\ -L^3_k L^k_3(1-\cos\theta) \\ -2L^2_k L^k_3(1-\cos\theta) \\ -2L^3_k L^k_1(1-\cos\theta) \\ -2L^1_k L^k_2(1-\cos\theta) \end{Bmatrix}, \quad k = 1,2,3 \tag{7}$$