Computers and Geotechnics xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

A long term evaluation of circular mat foundations on clay deposits using fractional derivatives

Cheng-Cheng Zhang [a], Hong-Hu Zhu [a,b,*], Bin Shi [a], Behzad Fatahi [c]

[a] School of Earth Sciences and Engineering, Nanjing University, Nanjing 210023, China
[b] Nanjing University High-Tech Institute at Suzhou, Suzhou 215123, China
[c] School of Civil and Environmental Engineering, University of Technology Sydney (UTS), Sydney, New South Wales 2007, Australia

ARTICLE INFO

Article history:
Received 7 June 2017
Received in revised form 31 July 2017
Accepted 24 August 2017
Available online xxxx

Keywords:
Ground settlement
Clay deposit
Foundation analysis
Fractional calculus
Time dependent behaviour
Settlement prediction

ABSTRACT

This study proposes to use fractional derivatives to evaluate the long term performance of circular mat foundations overlying clays and also predict the associated ground settlement. Closed form solutions for the deflection and bending moment of foundations and the subsequent reaction of subgrade are obtained with the Mittag–Leffler function. Numerical examples are used to determine how the fractional order affects the time dependent properties of the foundation and ground settlement, and to simulate the case history of a large standpipe constructed over Tertiary sediments. New insights into design and prediction of shallow foundations and ground settlement are also discussed.

1. Introduction

The performance of foundations constructed over clay deposits and the associated ground settlement are of great concern for geo-engineers [1–5] because the interaction between foundations and soil has hitherto been considered as a plate resting on a Winker or an elastic medium with two parameters. These early contributions did provide a sound theoretical basis for later study where complex loading histories and foundations with different shapes are considered [6–10], analytical and numerical methods such as the point collocation method, the Hankel transform, the ring method, the Chebyshev polynomial–Ritz method, and the finite element method have also been proposed and utilised [11–16]. However, since the deformation of clay is time dependent, it should be considered when modelling the interaction between soil and structure. For decades, the performance of a foundation on a viscoelastic medium [17,18] or a poroelastic medium [19,20] has been investigated, which is why the theory of poroelasticity, proposed originally by Terzaghi and further developed by Biot, enables the dissipation of pore water pressure and deformation of the solid skeleton under external loads to be considered. While simple and convenient simulations of the viscous behaviour of clay using viscoelastic models (e.g. Maxwell model, Kelvin–Voigt model and Merchant model) can graphically visualise materials, they are still limited by the simple constitutive relationship of the Newtonian dashpot [21].

Since fractional calculus is where integrals and derivatives can be of arbitrary non-integer orders and it can simulate hereditary phenomena with a long memory [22], it was introduced to viscoelasticity in the first half of the 20th century [23], and since then, models based on fractional derivatives have been utilised extensively to describe the viscous properties of materials [24–27]. The concept of fractional calculus has been introduced to geotechnics [28–33] where the viscous behaviour of geomaterials such as clays [34], granular soils [35], saturated soils [36], frozen silts [37], and rocks [38], has recently been modelled. Interesting work by Cosenza and Korošak shows that the secondary consolidation of clay can be described using a time-fractional diffusion equation within the framework of Terzaghi's theory [34], but until now, research on the interaction between foundation and soil while incorporating fractional calculus is still limited [39,40]. While the time dependent response of a circular foundation has been investigated by assuming that the underlying soil is either viscoelastic [41] or poroelastic [42], the interaction between a circular foundation and a fractional type of viscoelastic soil has never been reported in literature.

* Corresponding author at: School of Earth Sciences and Engineering, Nanjing University, Nanjing 210023, China.
E-mail address: zhh@nju.edu.cn (H.-H. Zhu).

http://dx.doi.org/10.1016/j.compgeo.2017.08.018
0266-352X/

Nomenclature

C_α	coefficient of secondary consolidation
C_v	coefficient of primary consolidation
D	derivative
^{RL}D	Riemann–Liouville fractional derivative
D_f	bending rigidity of foundation
EOP	end of primary
E	elastic modulus of fractional derivative element
E_0	elastic modulus of fractional derivative Maxwell clay
E_f	elastic modulus of foundation
$\overline{E_0}$	analogous modulus of fractional derivative Maxwell model
$E_\alpha(\cdot)$	original Mittag–Leffler function
$E_{\alpha,1}(\cdot)$	generalised Mittag–Leffler function with one complex parameter equals 1
FDMM	Fractional derivative Maxwell model
$H(\cdot)$	Heaviside step function
h	thickness of foundation
$J_i(\cdot)$	Bessel function of the first kind of order i
M^*	non-dimensional moment of foundation
q_0	load
RMSE	root-mean-square error
R	reaction of subgrade
R^*	non-dimensional reaction of subgrade
r	radius
r_0	radius of foundation
r_1	radius of load
SMM	standard Maxwell model
S	ground settlement
S_i^{FDMM}	ith displacement calculated using FDMM
S_i^{meas}	ith displacement measured in field
s	parameter of Laplace transform
t	time
t_{EOP}	EOP consolidation time
w	deflection of foundation
w^*	non-dimensional deflection of foundation
$\bar{w}$	Laplace-transformed deflection of foundation
z	depth
α	order of fractional derivative
$[\alpha]$	smallest integer exceeding α
$\Gamma(\cdot)$	Euler gamma function
γ	non-dimensional parameter
ε	strain
η	coefficient of viscosity of fractional derivative element
η_0	coefficient of viscosity of fractional derivative Maxwell clay
μ	Poisson's ratio of foundation
v	order of fractional integration
ρ	non-dimensional radius
σ	stress
τ	creep time of fractional derivative element
τ_0	creep time of fractional derivative Maxwell clay
$\phi(t)$	function determining variation of load with time
$\overline{\phi}(s)$	Laplace-transformed function of $\phi(t)$

In this paper a fractional derivative Maxwell model is used to investigate the interaction between a circular mat foundation and clay deposits, and the associated ground settlement. First, analytical solutions for the deflection and bending moment of foundations and the reaction of subgrade are derived using the corresponding principle and the Laplace transform for an axisymmetric loading history. The solutions are then solved numerically and a comprehensive numerical example is given to show how the fractional derivative order affects the long term performance of foundations and time dependent ground settlement. The model's ability to simulate the interaction between a circular foundation and clayey ground is demonstrated by analysing the case history of a large water standpipe constructed on a circular mat foundation overlying Tertiary deposits. Finally, the implications of this research into the design of shallow foundations and predicting settlement are discussed.

2. Fractional derivative Maxwell model (FDMM)

2.1. Basic concepts in fractional calculus

In traditional calculus the nth derivative of a function $z(t)$ is defined as $D^n z(t) = d^n z(t)/dt^n$, but this function is extended if the integer order n is substituted by a fraction; this is the starting point for defining fractional calculus. In this study the Riemann–Liouville definitions of fractional integration and derivative are used; the Riemann–Liouville fractional integration of function $z(t)$ is given by [43]

$$ {}_0^{RL}D_t^{-v}z(t) = \frac{1}{\Gamma(v)}\int_0^t (t-\xi)^{v-1}z(\xi)\mathrm{d}\xi, \quad t > 0 \tag{1}$$

where D denotes derivation; v is the order of integration ranging from 0 to 1; and the subscripts 0 and t denote the integration limits. The Riemann–Liouville fractional derivative of order α is formulated as [43]

$$ {}_0^{RL}D_t^{\alpha}z(t) = {}_0^{RL}D_t^{[\alpha]}[{}_0^{RL}D_t^{-v}z(t)], \quad t > 0 \tag{2}$$

where $[\alpha]$ is the smallest integer exceeding α; and $v = [\alpha] - \alpha > 0$. This definition indicates that the fractional derivative has a strong inherent memory due to its integral form. For simplicity, the superscript "RL" and the subscripts 0 and t are dropped in the following sections.

2.2. Generalisation of the standard Maxwell model based on fractional derivatives

A fractional derivative element known as the "intermediate model" [25] or "spring-pot" [27], is introduced first. The constitutive law of this element is defined as

$$\sigma(t) = E\tau^{\alpha}D^{\alpha}\varepsilon(t), \quad 0 \leqslant \alpha \leqslant 1 \tag{3}$$

where σ and ε are stress and strain; E is the elastic modulus; τ is the creep time and equals η/E; and η is the coefficient of viscosity. Because the element collapses into a spring and a dashpot when α equals 0 and 1 respectively, the order of the fractional derivative α is a non-dimensional parameter associated with the memory of materials [27].

A standard Maxwell model (SMM) consists of a spring and a dashpot connected in series, but by replacing the dashpot with a fractional derivative element, the standard Maxwell model is upgraded to an FDMM (Fig. 1), whose stress–strain relationship reads

$$(D^{\alpha} + 1/\tau_0^{\alpha})\sigma(t) = E_0 D^{\alpha}\varepsilon(t) \tag{4}$$

where E_0, τ_0 and α are three independent parameters of the FDMM.