J. Non-Newtonian Fluid Mech. 165 (2010) 1020–1023

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: www.elsevier.com/locate/jnnfm

On kinetic theory methods in vehicular flow

A.R. Méndez [a,*], R.M. Velasco [b]

[a] Departamento de Matemáticas Aplicadas y Sistemas, Universidad Autónoma Metropolitana, 01120, Cuajimalpa, Mexico
[b] Departamento de Física, Universidad Autónoma Metropolitana, 09340, Iztapalapa, Mexico

ARTICLE INFO

Article history:
Received 24 November 2009
Received in revised form 28 January 2010
Accepted 22 February 2010

PACS:
89.40.−a
45.70.Vn
47.90.+a

Keywords:
Kinetic theory
Macroscopic traffic model
Traffic flow

ABSTRACT

Similarly to the treatment of diluted gases, kinetic methods are formulated for the study of unidirectional freeway traffic. From these it is possible to construct fluid-dynamic equations which in comparison with heuristic fluid-dynamic models have the advantage of just some adjustable parameters although they have some other restrictions. In this work the comparison between two macroscopic models which are based on a kinetic traffic equation is shown. On the other hand, there will be presented some advances to attempt to generalize some restrictions of the kinetic formulation in order to study the synchronization phenomena, which is a very interesting transitory phase between free flow and traffic jams.

1. Introduction

Kinetic equations have been used to construct hydrodynamic traffic models based on microscopic assumptions. In the kinetic theory of traffic has appeared a large number of kinetic equations to study unidirectional vehicular flow [1–6]. It is well known that macroscopic equations for averaged variables can be obtained from kinetic equations. To do this task, we need a microscopic model, a method to find an approximate solution in the kinetic equation and, a closure hypothesis. With such a squeme we have constructed two macroscopic models with a support based on the kinetic Paveri-Fontana equation [3]. Though the Paveri-Fontana equation was not the first kinetic model in the literature, it has been proven to overcome some of the shortcomings of the pioneer kinetic equation named as the Prigogine's model [1,2]. In this work we have used the Paveri-Fontana kinetic equation to construct macroscopic models assuming an aggressive drivers behavior [7–9]. Starting from a kinetic model, the macroscopic model thus obtained have some advantages and of course also some disadvantages. The principal one is that kinetic theory is restricted to the low density regime, and the macroscopic models derived from them selfrestrict to this region. This restriction has prevented us to apply such a model when we have the congested regime, hence in this work we generalize the model to a situation in which the experimental data give us the rule to go into the dense region. This paper is organized as follows. In Section 2 Prigogine and Paveri-Fontana models are presented and compared, and a particular solution for Paveri-Fontana's model is presented in Section 3. Sections 4 and 5 are devoted to macroscopic-kinetic models and their closure. In Section 6 numerical results are shown and finally in Section 7 we present some comments, conclusions and perspectives.

2. Kinetic equations

Kinetic theories are based on an equation for the distribution function f, where $f(x, v, t)\,dx\,dv$ denotes the number of vehicles at time t between x and $x + dx$ with speed between v and $v + dv$. Prigogine's model is the first one of this kind and was presented in 1960 [1,2]. Prigogine et al. stand the existence of a desired distribution function f_0, which is a mathematical idealization of how drivers desire collectively to drive. Prigogine and co-workers assumed that the distribution function may deviate from the desired distribution function f_0 due to various factors, e.g. road factors, weather conditions or interactions with other vehicles. On such situations and provided interactions are negligibly small, f relaxes to f_0 over a constante relaxation time τ, although it may be a function of density or some other variable. Based on these assumptions and in analogy to the Boltzmann kinetic equation, Prigogine et al. suggested the next

* Tel. +52 55 2636 3800.
E-mail addresses: armr@xanum.uam.mx (A.R. Méndez), rmvb@xanum.uam.mx (R.M. Velasco).

0377-0257/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.jnnfm.2010.02.018

kinetic equation

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}=\underbrace{-\frac{f-f_0}{\tau}}_{\text{relaxation}}+\underbrace{f\int_0^\infty \mathrm{d}v'(1-p)(v'-v)f'}_{\text{interaction}}. \tag{1}$$

where the first term to the right accounts for a relaxation of f towards f_0 and the second term accounts for the changes in f due to interactions between vehicles. For the first term, Prigogine proposed a collective relaxation toward f_0 in a single relaxation time τ, where the desired speed distribution remains independent of the local concentration $\rho(x,t)$ so that $f_0(x,v,t)=\rho(x,t)F_0(v)$. In the interaction term are considered the increase/decrease of the number of vehicles in the phase-space due to interactions between vehicles. The concept of desired distribution as well as the scenario of collective relaxation, in a diluted system, has been received severe criticism [3] and very commonly this model has been replaced by the Paveri-Fontana model.

To overcome the problems of Prigogine's model, Paveri-Fontana proposed an alternative model where he introduces a new phase-space coordinate, the so called desired velocity w. In this case, the distribution function $g(x,v,w,t)\,\mathrm{d}x\,\mathrm{d}v\,\mathrm{d}w$ denotes the number of vehicles at time t, in position $\mathrm{d}x$ around x, velocity dv around v and desired velocity $\mathrm{d}w$ around w. With $\vec{x}=(x,v,w)$, the total local change of the phase space density is given through a continuity equation

$$\frac{\partial g}{\partial t}+\nabla_{\vec{x}}\cdot\left(g\frac{\mathrm{d}\vec{x}}{\mathrm{d}t}\right)=\left(\frac{\partial g}{\partial t}\right)_{\text{int}},$$

where the second term in the right hand side represents the drift in the correcponding phase space and in the right hand side we have the contributions coming from the interaction between vehicles. For the interaction term Paveri-Fontana proceded similarly as Prigogine. The drift term was constructed taking into account the following ideas: for the acceleration term he proposes $\mathrm{d}v/\mathrm{d}t=(w-v)/\tau$, i.e. the drivers approach their desired speed exponentially in time, with an individual relaxation time τ and additionally, it is considered that drivers do not changes their desired speed thus ($\mathrm{d}w/\mathrm{d}t=0$). In this way the Paveri-Fontana equation (PFE) reads

$$\begin{aligned}\frac{\partial g}{\partial t}+v\frac{\partial g}{\partial x}+\underbrace{\frac{\partial}{\partial v}\left(\frac{w-v}{\tau}g\right)}_{ind.rel.}\\ =f(x,v,t)\int_v^\infty \mathrm{d}v'(1-p)(v'-v)g(x,v',w,t)-g(x,v,w,t)\\ \times\int_0^v dv'(1-p)(v-v')f(x,v',t),\end{aligned} \tag{2}$$

where the right hand side represents the interaction between vehicles and

$$f(x,v,t)=\int \mathrm{d}w \quad g(x,v,w,t).$$

In order to compare with Prigogine's Eq. (1) it is possible to integrate Eq. (2) over all w's to obtain the reduced Paveri-Fontana equation

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}+\frac{\partial}{\partial v}\left(f\frac{W(x,v,t)-v}{\tau}\right)=f\int_0^\infty(1-p)(v'-v)f'\,\mathrm{d}v', \tag{3}$$

where $f(v,x,t)W(v,x,t)=\int_0^\infty wg(x,v,w,t)\,\mathrm{d}w$.

In analogy to the scattering process in kinetic gas theory, both kinetic models have the usual restrictions, see [3]. To compare the Paveri-Fontana equation with Prigogine equation observe that the central difference is the introduction of $g(x,v,w,t)$, which involves a new variable in phase space instead of assuming the existence of a desired distribution function f_0. As can be seen, both cases consider the interaction process described in the same way. In contrast, the relaxation process, though based in the existence of a unique relaxation time, has different physical meaning. In Prigogine's model it represents a collective relaxation towards the velocity distribution f_0 and, in Paveri-Fontana's equation the relaxation time corresponds to an individual relaxation. This treatment permits to bypass the problem of assigning f_0 *a priori* and the problems pointed out by Paveri-Fontana in his thought experiments [3].

3. Steady and homogeneous solution

If a simple model for aggressive drivers is considered, it is possible to obtain an analytical solution of the reduced PFE for the steady and homogeneous state (SHS), usually called as the equilibrium state. The details of this calculations can be seen in [8,9], where is proposed a model for the desired velocity $W(x,v,t)=\omega v$, and ω is a proportionality constant bigger than unity but close. This model is called for aggressive drivers cause the desired velocity is modeled proportional to actual speed, which means than in average the drivers want to drive a little faster than they are driving. Assuming this model, the solution of (3) for the the steady and homogeneous state is

$$f_e(v)=\frac{\alpha}{\Gamma(\alpha)}\frac{\rho_e}{V_e}\left(\frac{\alpha v}{V_e}\right)^{\alpha-1}\exp\left(-\frac{\alpha v}{V_e}\right), \tag{4}$$

where

$$\alpha=\frac{(1-p(\rho_e))\rho_e V_e\tau}{\omega-1}, \tag{5}$$

is a constant which may be determined by the experimental data. In fact, the fundamental diagram $V_e(\rho_e)$, the probability of overpassing $p(\rho_e)$ and, the relaxation time are quantities for which some values are available. On the other hand, it is possible to verify that the quantity α is closely related with the variance prefactor

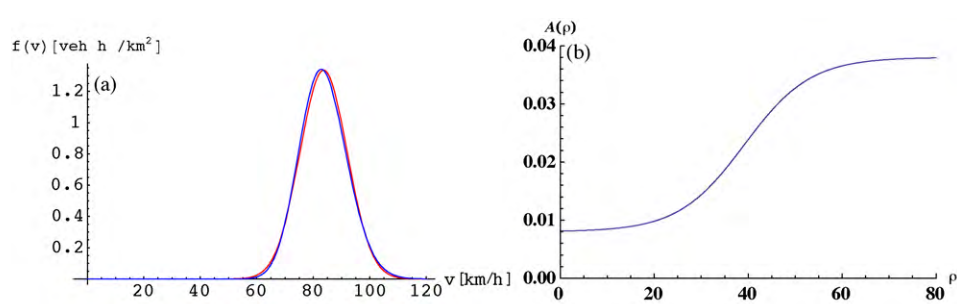

Fig. 1. (a) Comparison of the distribution function (4) (blue) and a gaussian distribution (red) for the same SHS values. (b) $A(\rho)$ proposed by Helbing in [11].(For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.).