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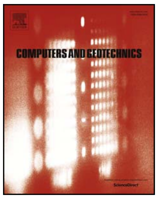

Research Paper

A stress integration scheme for elasto-plastic response of unsaturated soils subjected to large deformations

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ABSTRACT

This paper proposes an algorithm for performing explicit integration of constitutive models for unsaturated soils in problems where consideration of stress rate objectivity becomes necessary due to large deformations. As many constitutive models for unsaturated soils incorporate suction forces, as well as stresses and strains, into the material description, an accurate integration procedure is more complicated than similar schemes proposed for saturated soils. The difficulties of integration are also coupled with the need to incorporate objectivity (frame independency) within the integration procedure. Application of the proposed method is limited to constitutive models that assume additive decomposition of the strains. A fully coupled finite deformation numerical model is considered with displacement, pore water pressure and suction as the nodal variables. Details of the implementation of the stress integration algorithm at each stage of an analysis are given. The algorithm is then applied to solve some large-scale geotechnical problems. Numerical examples are also provided to assess the performance of the algorithm under different loading conditions.

1. Introduction

The governing equations of solid and fluid interaction in a porous medium were first developed for quasi-static situations by Biot [6], and were later extended to dynamic problems by Biot [7]. The subsequent introduction of 'mixture' theory by Truesdell [75], and improvements to this theory by researchers such as Green and Naghdi [24], Barden et al. [4] and Bowen [11], paved the way for the establishment of a new basis for thermo-hydro-mechanical-chemical analysis of porous media, allowing for the existence of multiphase pore fluids. This made it possible to incorporate some advanced features of multi-phase material response, including phase changes, chemical reactions, and behaviour under a non-isothermal environment. The extension of Biot's theory to unsaturated soils was presented by Li et al. [36] and Li and Zienkiewicz [38], who assumed that no phase transfer nor any chemical reactions were possible during the fluid flows. This kind of flow is called "immiscible" and is the subject of this research. There are several numerical approaches for solving the equations governing the behaviour of unsaturated porous media. Morel-Seytoux and Billica [43] and Wu and Forsyth [79] studied the problem of multi-phase flow inside a rigid porous body with no deformation in the solid phase. However, by using a series of tests on an unsaturated sand column, Narasimhan and Witherspoon [44] showed that agreement with experimental results cannot be achieved unless deformation of the soil skeleton is also taken into account. In this paper, the behaviour of an unsaturated porous medium is studied using a fully dynamic and coupled solution method, based on the previous works of Li et al. [39], Li and Zienkiewicz [38], Schrefler and Xiaoyong [61], Schrefler et al. [63], Li et al. [37], Rahman and Lewis [56], Schrefler and Scotta [62], Ehlers et al. [18], Nuth and Laloui [50] and Khoei and Mohammadnejad [34].

2. Effective stress in the analysis of unsaturated soils

The choice of appropriate stress variables is a key component in constructing a coupled formulation for modelling the response of unsaturated porous media. It provides the capability of extending constitutive models developed for saturated soils to partially fluid-filled geomaterials.

The effective stress definition suggested by Bishop [8] for unsaturated soils is adopted here, i.e.,

$$\sigma'_{ij} = \sigma_{ij} + p_w\delta_{ij} + (1-\chi)p_c\delta_{ij} \quad (1)$$

where σ'_{ij} is the effective stress; σ_{ij} represents the total stress; p_c denotes the suction; p_w is the pore water pressure which is considered positive

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in compression in this paper; χ is called the effective stress parameter or Bishop's parameter, which ranges from 0 to 1, with the extreme values corresponding to dry and saturated conditions, respectively; and δ_{ij} is the Kronecker delta.

Jennings and Burland [29] were among the first to raise doubts about the validity of this definition, pointing out that it is incapable of explaining the process of soil compression on wetting (known as collapse). This argument is true only if a single effective stress based on Bishop's proposition is adopted when extending constitutive models developed for fully saturated soils to unsaturated soils. Subsequently, Khalili et al. [32] showed that such collapse behaviour is actually linked to a plasticity mechanism. As noted by Manzanal et al. [41], the collapse phenomenon can be captured by using the effective stress defined in Eq. (1) if it is accompanied by an internal state parameter such as suction. This approach is now adopted in most constitutive models based on the concept of effective stress, such as those proposed by Santagiuliana and Schrefler [60], Russell and Khalili [59] and Manzanal et al. [41].

In this paper the same approach is chosen, with the degree of fluid saturation considered as Bishop's parameter. Indeed, this representation of effective stress can also be found in Bolzon et al. [9], Lewis and Schrefler [35], Hutter et al. [28], Jommi [31], Wheeler et al. [78], Ehlers et al. [18], Tamagnini [73], Oka et al. [51], Nuth and Laloui [50] and Khoei and Mohammadnejad [34], with different names being adopted, such as for example the "skeleton stress". It has been suggested by Houlsby [25] that making this choice is justified in terms of the effective stress so defined being work conjugated. However, subsequently, Houlsby's approach was shown by Gray and Schrefler [23] to suffer from some shortcomings, in particular, in ignoring the work being undertaken along the phase interfaces.

The dependency of the Soil Water Characteristic Curve (SWCC) on the volume changes of the soil and its hysteretic behaviour have been specifically ignored here (Appendix D), in order to keep the definitions of stress and strain as simple as possible. Indeed, Houlsby [25] and Jiang et al. [30] showed that the key pair of parameters we have adopted, i.e., the degree of saturation (Bishop's parameter) and the suction, is thermodynamically consistent if the dependency of the soil water characteristic on the volume changes of the soil is ignored. It is worth mentioning that other alternative definitions of the effective stress for unsaturated materials, such as those proposed by Khalili and Khabbaz [33] or more recently by Alonso et al. [2] and Jiang et al. [30], can also be considered in the analysis of unsaturated porous media, particularly, if the dependency of the soil water characteristic curve on the volume changes is considered.

An example of the use of an alternative Bishop's parameter in the finite element framework, based on the equation proposed by Khalili and Khabbaz [33], can be found in Shahbodagh-Khan et al. [64] and Tang et al. [74] who also considered the dependency of the SWCC to the volume change and its hysteretic response in their analyses. Nonetheless, simplifications made to the SWCC equation would not affect the stress integration suggested later in this paper.

3. Numerical integration schemes for elasto-plastic constitutive models

In most constitutive models for geomaterials the rate of stress is usually expressed as a function of the rate of strain. Hence, obtaining the updated stress values in each step requires the integration of the constitutive equations over the specified strain intervals. It is generally impossible to integrate these constitutive models analytically due to their complexity, and robust numerical integration methods are required to approximate the exact solution. However, these methods, if incautiously chosen, may also impose additional complexity on the implementation procedure and, most importantly, can negatively affect the performance of the constitutive equations in actual computations, as mentioned by Hughes [26]. Conversely, a robust and efficient numerical procedure can simplify the implementation of constitutive models into a finite element program greatly, thus facilitating the analysis of large-scale, complex geotechnical problems. Although many numerical integration algorithms have been developed over the last four decades, these can be broadly classified into two main categories: explicit and implicit algorithms.

Generally, explicit methods are frequently used because of their simplicity of implementation and robustness. On the other hand, implicit integration schemes, in particular for the case of models with non-convex yield surfaces, have often been cited as having significant advantages over explicit approaches in dealing with large strain increments; thereby increasing the speed of analysis. However, most implicit methods, such as the closest point projection method (Simo and Taylor [68]), often exhibit convergence problems in the following situations (Anandarajah [3]):

- When the initial guess for the solution (required at the start of the algorithm) is far from the correct solution. This can occur frequently in highly nonlinear elasto-plastic models with softening and hardening behaviour.
- In models based on Cam-Clay, the hardening parameter (determining the yield surface size) may experience a change from a positive value to a physically inappropriate negative value during an iteration. Such an occurrence, if not detected and appropriately managed, usually causes termination of the analysis.
- In advanced constitutive models for geomaterials that describe the elastic and plastic moduli as functions of the mean normal pressure, numerical instability may occur when the magnitude of the mean normal pressure is less than some arbitrarily small value. This may result in the plastic modulus varying from a positive to a negative value, which again can cause convergence difficulties.

Various research papers have proposed solutions to enhance the performance of implicit algorithms, such as those by de Souza Neto et al. [16] and Pérez-Foguet et al. [52]. However, including such enhancements adds to the complexity of the implementation and may have undesirable consequences. For example, the important contribution of Pérez-Foguet et al. [52] always leads to non-symmetric systems of global equations, even for the case of a plasticity model with an associated flow rule. Also, if Newton iteration is used in the implicit integration algorithm, finding the second derivatives of the yield function and the plastic potential becomes necessary. This adds both expense and complexity to the implementation.

This paper combines a sub-stepping technique with an explicit integration method for an unsaturated soil model, where an objective stress rate is needed for large deformation analysis. First, the governing equations and the constitutive model employed are presented briefly. Then, the integration algorithm and its ingredients are explained in detail. Finally, the algorithm is applied to various unsaturated soil problems involving large deformations.

4. Governing equations and the constitutive model

In this study, the global equations describing the time-dependent deformation of an unsaturated porous medium, loaded either statically or dynamically, are discretised using the generalised-α method (Chung and Hulbert [14]). The discretisation details and the validation of the model have been presented previously by Ghorbani et al. [21], Ghorbani et al. [21], Ghorbani et al. [22] and Ghorbani [20]. A summary of the equations is given in Appendix A. To consider the effects of large deformations, the modifications due to the Updated Lagrangian (UL) framework are detailed in Appendix B.

The adopted soil model, suggested by Sheng et al. [66], Sheng et al. [67], is an extension of the Modified Cam Clay (MCC) model first proposed by Roscoe and Burland [58] for saturated soils. An associated plastic flow rule is assumed where the yield surface, f, and the plastic