Computers and Geotechnics 81 (2017) 49–58

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Hybrid particle swarm optimization for first-order reliability method

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ARTICLE INFO

Article history:
Received 7 April 2016
Received in revised form 24 June 2016
Accepted 28 July 2016

Keywords:
Reliability
First-order reliability method
Hybrid particle swarm optimization
Rock foundation of gravity dam

ABSTRACT

A reliability analysis method based on the combination of the first-order reliability method (**FORM**) and hybrid particle swarm optimization (**SACPSO**) is presented for the reliability optimization calculation. The new reliability method, named as **SACPSO-FORM**, can be utilized for those complex reliability problems with correlated non-normal variables and implicit performance functions. Three examples are performed to verify its validation, and stability reliability analysis on the complicated rock foundation of a practical gravity dam is demonstrated. The results show that the proposed method is accurate, stable, flexible and efficient for reliability analysis in engineering applications.

1. Introduction

It is a trend to use the reliability method for evaluation of the risk of failure in almost all engineering fields, including some in geotechnical engineering [1–3]. The current challenge to the engineering profession is to carry out reliability theory in practice [4]. Many researchers have focused on the research topics of reliability-based design and risk analysis, and made progress in resolving the problem about the failure of slopes, levees, embankments, dams and other geotechnical engineering in recent years [1,4,5].

The approximate methods to estimate the reliability index β for time-invariant reliability analysis in geotechnical engineering can be basically classified to five categories as follows: (1) sampling method such as Monte Carlo simulation (**MC**) [6]; (2) the most probable point (**MPP,** or design point) based method such as the first order reliability method (**FORM**) [1,7] and second order reliability method (**SORM**) [7–10]; (3) expansion method such as first-order second-moment method (**FOSM**) [1,5]; (4) response surface method (**RSM**) [11–17]; (5) approximate integration method [18,19]. In fact, every method mentioned above has its merits and limitations in terms of accuracy, efficiency, robustness and flexibility. **MC** is an accurate and robust method to estimate β for almost all kinds of reliability problems when sufficient large sample size is used, while it is rarely adopted due to its huge calculation time for small probability failure problems or problems with complicated performance function, $g(\boldsymbol{X})$ [6]. **FOSM** is an efficient numerical method to obtain β, but the applicability is relatively limited because the partial derivatives of $g(\boldsymbol{X})$ are difficult to be derived from implicit to explicit involving complex failure modes of geotechnical engineering at some times [1,2]. **RSM** and improved **RSM**s are usually utilized to obtain the solution of β for complex reliability problems, but it is relatively complicated and also needs much more computing cost with lower efficiency, since it has a lot of iterating calculation at different design points associated with numerical method (i.e., finite element method) to fit limit state curved surface, $g(\boldsymbol{X}) = 0$ [11–17]. As to the approximate integration method, it is always not suitable to solve the geotechnical reliability problems with complex or implicit $g(\boldsymbol{X})$ [19].

FORM and **SORM** are based on first-order or second-order approximations of the limit state at the **MPP** of failure (or design point). **SORM** was developed by Kiureghian and his colleagues [7,8]. Many others have proposed numerical techniques for solving the underlying mathematical problem of the **SORM**, such as asympotic approximation for multinormal integrals developed by Breitung and his coauthors at first [20,21]. However, the nonlinear and explicit performance function, $g(\boldsymbol{X})$, for **SORM** must be known previously, and the tedious Hessian matrix must be solved during calculation. So **SORM** has not been widely used in geotechnical reliability applications, especially for the reliability analysis with correlated non-normal variables and implicit $g(\boldsymbol{X})$. As well known, **FORM** is widely used in reliability analysis due to its simplicity, efficiency and flexibility, and was recommended by the Joint Committee of Structural Safety (**JCSS**). The basic theory of **FORM** was developed by Hasofer and Lind [22]. Rackwitz [23] indicated that for 90% of all applications with respect to time-invariant reliability

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http://dx.doi.org/10.1016/j.compgeo.2016.07.013
0266-352X/

computation this simple method fulfills the practical needs. The traditional **FORM** is usually expressed as the minimization of objective function associated with β subjected to limit state, $g(\boldsymbol{X}) = 0$ [24–28]. For traditional **FORM**, the performance function must be explicit in most cases. For the case of implicit performance function, some techniques have been developed. For example, Low and Tang successively developed spreadsheet-cell-object-oriented constrained optimization approach and another efficient spreadsheet algorithm for reliability analyses [29,30]. Low and Tang's methods can process the reliability problems involving non-normal and correlated variables and explicit or implicit performance function. However, the traditional **FORM** and Low and Tang's methods utilize the gradient based method for convergence and easily fall into local optimal solution of β. So **FORM** is confined from complicated reliability computation and a suitable optimization searching procedure is critical for the global solution. Based on the principle of **FORM**, many researchers have tried to use other optimization algorithms for the reliability optimization calculation [31,32]. Cheng applied a hybrid genetic algorithm based **FORM** to structural reliability analysis [32].

In the paper, a novel **FORM** based on a hybrid intelligent evolutionary algorithm, **SACPSO**, which combines particle swarm optimization with constriction coefficient (**CPSO**) and simulated annealing process (**SA**), is proposed to solve optimization problems of time-invariant reliability analyses in geotechnical engineering. The article is organized as follows. In Section 2, the proposed **SACPSO-FORM** is introduced in detail. In Section 3, the validation of the method is demonstrated by 3 different types of examples for reliability analyses and then compared with other conventional reliability methods, such as **MC**, **RSM**, **FOSM** and traditional **FORM**, regarding solution quality and computational efficiency. In Section 4, the **SACPSO-FORM** is applied to realize the optimum calculation of stability reliability index of complicated rock foundation over multiple sliding planes for an actual gravity dam under construction, and the results are compared with **MC**. We draw some conclusions in Section 5.

2. Method

2.1. FORM method

For **FORM** [7–10], the reliability index β subject to the constrain is

$$\begin{aligned} &\beta = \min\left[(X-\mu)^{\mathrm{T}}[C_X]^{-1}(X-\mu)\right]^{1/2} \\ &ST.Z = g(X) = 0 \end{aligned} \tag{1}$$

where X is the vector of uncertain variables; μ and σ are respectively the vectors of the means and standard deviations of X; $\boldsymbol{C}_X$ is covariance matrix; $g(X)$ is the performance function.

The objective function $f(X)$ for reliability calculation can be expressed as $f(X) = \left[(X-\mu)^{\mathrm{T}}[C_X]^{-1}(X-\mu)\right]^{1/2}$ (see the first line of Eq. (1)). If $f(X)$ equals a constant, it will be described as an ellipsoid in terms of multivariate X in matrix format. In Fig. 1, for example, the 1-σ ellipse, the β-σ ellipse, and the failure surface are plotted together for a 2-dimension reliability problem when correlation coefficient $\rho = 0.7$. The ellipse that is tangent to the failure surface, $g(\boldsymbol{X}) = 0$, is β times the size of the 1-σ dispersion ellipse. This provides a geometric interpretation of the reliability index β in the original space of the random variables. The point of tangency, P^*, is the design failure point represented by (X_1^*, X_2^*, ..., X_n^*) which can be found by optimization.

When the random variables are correlated and in non-normally distribution, the Rackwitz-Fiessler equivalent normal transformation is implemented without considering to diagonalize the covariance matrix $\boldsymbol{C}_X$ [24,33,34].

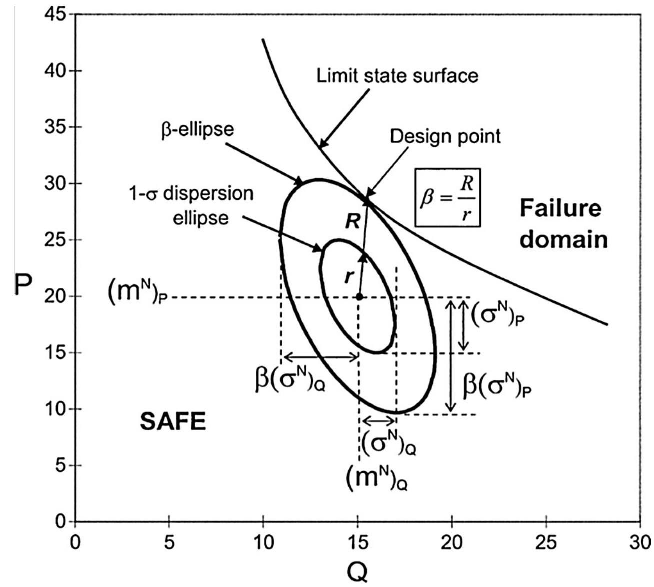

Fig. 1. 1-σ dispersion ellipse and critical ellipse of some a 2-dimension reliability problem (Low and Tang, 2004) [29].

2.2. Hybrid particle swarm optimization

2.2.1. Basic *PSO*

Particle swarm optimization (**PSO**) has been used to tackle optimization issues in the past decade. It is in the form of probabilistic heuristics with global search properties. The algorithm was first proposed by Kennedy and Eberhart [34], and subsequently its convergence rate and method to select the best parameters had been discussed [35]. **PSO** applies swarm intelligence to obtain the goal of optimization. The population refers to the best experience of the individuals and group, respectively, and some evolutionary methods by which individual will move itself by the experience are logically applied. After continuous iterations, the particle population will concentrate towards the optimum solution rather than randomly looking for the solution. The significant characteristic of **PSO** is in its simple structure, fast convergence, and its ability to prevent falling into a local optimum solution [36–42].

In the basic **PSO** algorithm, each particle is in the d-dimensional search space. The position vector of the ith particle is recorded as $s_i = [s_i^1, s_i^2, \ldots, s_i^k, \ldots, s_i^d]$ and its velocity vector is represented as $v_i = [v_i^1, v_i^2, \ldots, v_i^k, \ldots, v_i^d]$. In each generation j, the particles are manipulated according to the following equations:

$$v_{i,j+1}^d = v_{i,j}^d + c_1 \cdot rand_1 \cdot (pbest_{i,j}^d - s_{i,j}^d) + c_2 \cdot rand_2 \cdot (gbest_j^d - s_{i,j}^d) \tag{2}$$

$$s_{i,j+1}^d = s_{i,j}^d + v_{i,j+1}^d \tag{3}$$

where c_1 and c_2 are learning factors of **PSO**; $rand_1$ and $rand_2$ are random numbers between 0 and 1; $pbest_{i,j}^d$ is individual best optima for particle i after j iterations; $gbest_j^d$ is group optima after j iterations. Particle's velocity, $v_{i,j}^d$, in each generation on each dimension is confined to a maximum velocity parameter $v_{\max}$, specified by the users [35].

Kennedy presented that the trajectories of non-stochastic one-dimensional particles contained interesting regularities when $c_1 + c_2$ was between 0.0 and 4.0 [35]; Clerc and Kennedy's analysis of the iterative system led them to provide a strategy for the placement of "constriction coefficients" in the terms of the above formulas [36]. This adjusted **PSO** is called **CPSO** in the paper. The