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Laminar natural convection from a heated square cylinder immersed in power-law liquids

C. Sasmal, R.P. Chhabra *

Department of Chemical Engineering, Indian Institute of Technology, Kanpur 208016, India

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ABSTRACT

Laminar natural convection heat transfer from a heated long cylinder of square cross-section submerged in stagnant power-law fluids has been investigated numerically. The governing differential equations (continuity, momentum and thermal energy) have been solved over wide ranges of the pertinent dimensionless parameters, namely, Grashof number ($10 \leqslant Gr \leqslant 10^5$), Prandtl number ($0.72 \leqslant Pr \leqslant 100$) and power-law index ($0.3 \leqslant n \leqslant 1.8$) thereby covering both shear-thinning and shear-thickening type fluid behaviours. Detailed structure of the flow is studied in terms of streamline and isotherm patterns while heat transfer characteristics are analyzed in terms of the local Nusselt number distribution over the surface of the cylinder as well as its surface averaged values. Broadly, the flow remains attached to the surface up to larger values of the Grashof number in shear-thinning fluids ($n < 1$) than that in Newtonian media ($n = 1$). Similarly, all else being equal, shear-thinning behaviour promotes heat transfer. Indeed, it is possible to enhance the rate of heat transfer by up to 100% under appropriate conditions, i.e., values of the Grashof number, Prandtl number and power-law index. Of course, shear-thickening fluid behaviour has an adverse influence on the rate of heat transfer. In the limiting case of the Newtonian fluid behaviour ($n = 1$), the present predictions are in excellent agreement with the scant experimental results available in the literature.

1. Introduction

Heat transfer from a square cylinder denotes an idealization of many industrial applications. Typical examples include design of novel heat exchanger, cooling of electronic components, thermal treatment of various food-stuffs like potato and carrot chips, beans, etc., fluidized bed drying of fibrous substances and convective drying of wood and ply boards, etc. In addition to such an overwhelming pragmatic significance, this flow configuration is also frequently employed to gain useful insights into the nature of the underlying physical processes involving momentum and heat transfer and wake phenomena, etc. Consequently, over the years, a significant body of knowledge has accrued on momentum and heat transfer characteristics in Newtonian fluids from cylinders of various cross-sections including circular, elliptical, triangular and square [1,2]. It is also appropriate to add here that not only most of such studies are limited to air and water, but more importantly these pertain to either the forced or mixed convection regime of heat transfer [3,4]. Obviously, in most applications, free or natural convection always contributes in varying proportions to the overall rate of heat transfer, and indeed there are numerous situations in food processing applications where the only mode of heat transfer is by natural convection [5–7]. Typical examples include heating of canned foods, and of other liquids confined in closed spaces. Furthermore, even in mixed convection regimes, the role of natural convection gradually increases with the decreasing magnitude of the imposed flow, i.e., the decreasing value of the Reynolds number. In spite of its practical and theoretical importance, free convection in quiescent fluids from a long heated square cylinder has received very little attention, even in Newtonian fluids like air and water [8]. In addition to the physical properties of the medium and the size of the submerged cylinder, the rate of heat transfer is also influenced by the orientation of a square cylinder (horizontal, vertical or inclined) with respect to the direction of the gravity vector. Indeed, the corresponding values of the heat transfer coefficient can vary significantly with orientation under otherwise identical conditions. Furthermore depending upon the value of the Reynolds number (based on the characteristic velocity due to buoyancy-induced flow), the flow exhibits different flow regimes like laminar or turbulent, steady or unsteady, etc. This work is concerned with the prediction of heat transfer from a horizontal cylinder of square cross-section to quiescent ambient medium in the laminar flow regime. It is perhaps appropriate to add here that based on a combination of approximate analytical/numerical solutions complemented by experimental measurements, an adequate body of knowledge is now available enabling the prediction of the rate of heat transfer from a square cylinder in Newtonian fluids, at

* Corresponding author. Tel.: +91 512 2597393; fax: +91 512 2590104.
E-mail address: chhabra@iitk.ac.in (R.P. Chhabra).

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Nomenclature

C_D	drag coefficient, $C_D = \frac{2F_D}{\rho U_c^2 D}$, dimensionless
C_{DF}	frictional component of drag coefficient, $C_{DF} = \frac{2F_{DF}}{\rho U_c^2 D}$, dimensionless
C_{DP}	pressure component of drag coefficient, $C_{DP} = \frac{2F_{DP}}{\rho U_c^2 D}$, dimensionless
C	thermal heat capacity of fluid, J kg^{-1} K^{-1}
C_p	pressure coefficient, dimensionless
D	side of the cylinder, m
D_∞	diameter of the outer domain, m
F_D	drag force per unit length of the cylinder, N m^{-1}
F_{DF}	frictional component of drag force per unit length of the cylinder, N m^{-1}
F_{DP}	pressure component of drag force per unit length of the cylinder, N m^{-1}
g	acceleration due to gravity, m s^{-2}
Gr	Grashof number, dimensionless
h	heat transfer coefficient, W m^{-2} K^{-1}
I_2	second invariant of the rate of the strain tensor, s^{-2}
k	thermal conductivity of fluid, W m^{-1} K^{-1}
m	power-law consistency index, Pa s^n
n	power-law index, dimensionless
n_s	unit normal vector
Nu	average Nusselt number, dimensionless
Nu_l	local Nusselt number, dimensionless
N_P	number of grid points on each side of the cylinder
P	pressure, Pa
p	local pressure on the cylinder surface, Pa
p_∞	reference pressure far away from the cylinder, Pa
Pr	Prandtl number, dimensionless
Ra	Rayleigh number ($= Gr \cdot Pr$), dimensionless
S	surface area, m^2
T	temperature of fluid, K
T_w	cylinder surface temperature, K
ΔT	temperature difference ($= T_w - T_\infty$), K
T_∞	ambient fluid temperature, K
U_c	characteristic or reference velocity, m s^{-1}
U_x, U_y	x- and y-components of the velocity, m s^{-1}
x, y	cartesian co-ordinates, m

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β	coefficient of volume expansion, K^{-1}
ε	components of the rate of the strain tensor, s^{-1}
η	viscosity, Pa s
ν	kinematic viscosity, m^2 s^{-1}
θ	non-dimensional temperature, $\theta = \frac{T - T_\infty}{T_w - T_\infty}$
ρ	density of the fluid, kg m^3
τ	extra stress tensor, Pa

Subscripts

∞	ambient condition
w	cylinder surface condition

least in the laminar steady flow regime. However, there is a preponderance of studies relating to air as the ambient medium (i.e., one value of Prandtl number of 0.7–0.72) [8,9]. Thus, there is a paucity of information in this field as compared to that when heat transfer occurs in forced or mixed convection regimes from a square cylinder, e.g., [10–18].

On the other hand, it is readily acknowledged that many materials of multiphase nature (like foams, suspensions, emulsions, for instance), of high molecular weight polymers and surfactants/soap solutions, etc. all exhibit a range of non-Newtonian flow characteristics including shear-thinning, shear-thickening, visco-elasticity, etc. under appropriate concentrations and/or flow conditions [19,20]. Such structured systems are frequently encountered in food, pharmaceutical, polymer and personal care products manufacturing sectors, for instance [19–21]. In spite of their wide occurrence, only limited work is available on free convection heat transfer from a square cylinder, especially in non-Newtonian media [10]. This work endeavours to alleviate this situation by elucidating the influence of shear-thinning and shear-thickening viscosity characteristics on free convection heat transfer from a square cylinder. However, prior to presenting the new results obtained here, it is deemed to be instructive and useful to briefly review the pertinent limited literature on free convection from a square cylinder in Newtonian fluids and on momentum and heat transfer characteristics in non-Newtonian fluids. This, in turn, facilitates the subsequent presentation and discussion of the new results gleaned in this study.

2. Previous work

From a theoretical standpoint, the momentum and thermal energy equations describing the phenomenon of free convective transport from a square cylinder submerged in quiescent fluid media are required to be solved simultaneously due to their coupled nature via the body force term. Therefore, the complete solutions applicable over all ranges of the governing parameters, namely, Grashof number and Prandtl number have been rather hard to come by. This is also so even for the simple case of a long cylinder or a sphere [21–23]. The early attempts in this field are based on the application of the standard boundary layer approximations which are really valid for large values of Grashof number or of Prandtl number or both. In spite of this inherent limitation, this approach does lead to reasonable scaling of the Nusselt number with Grashof and Prandtl numbers, or the Rayleigh number for a cylinder or a sphere. On the other hand, a few workers have treated the free convection heat transfer from a square cylinder by decomposing it into vertical and horizontal plane surfaces. Naturally, most of these approaches have met with varying degrees of success in correlating the available limited data on free convection heat transfer from a horizontal square cylinder [24,25]. Similarly, a few attempts have been made to identify appropriate shape factors and characteristics linear dimensions for two- and three-dimensional objects to develop unified correlations for free convection heat transfer which purport to be applicable over wide ranges of conditions [25,26].

Chang et al. [29] appeared to be the first who numerically solved the complete governing equations for laminar natural convection heat transfer from a horizontal heated square cross-section cylinder placed in a large body of stagnant air. Since it is not possible to simulate truly unconfined flow condition, they used a relatively short (eight times the cylinder size) cylindrical domain with the heated cylinder located at its centre. No flow separation was observed up to about Rayleigh numbers of $\sim 10^3$. This finding is qualitatively in line with the experimental observations of Cho and Chang [28]. Subsequently, this configuration with identical flow domain has been re-visited over a somewhat wider range of the Rayleigh number as $10^3 \leqslant Ra \leqslant 10^5$ for a fixed value of $Pr = 0.7$ corresponding to air [29]. The resulting values of the Nusselt number were found to be in agreement with the available experimental data. More recently, Zeitoun and Ali [30] have numerically investigated free convection from heated two-dimensional cylinders of