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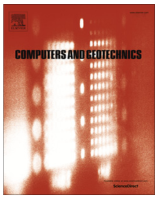

Modelling comminution of granular materials using a linear packing model and Markovian processes

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ABSTRACT

This paper proposes an analytical model to calculate the evolution of the grain size distribution due to crushing of granular materials under cyclic loading. The model, that requires very low computational cost, combines a theory for polydisperse mixtures with Markovian processes. The evaluation of the performance of the model was carried out experimentally using a gyratory compactor. The comparison of the results obtained experimentally with those obtained using the proposed model shows very good agreement. As a result, the analytical model proposed here is a useful alternative to be included in a model based in continuum mechanics.

1. Introduction

Particle fracture due to cyclic loading taking place in transportation works such as roads, railroads, or embankments affect the most important engineering properties of granular materials and, therefore, affect the long term performance of these works. Assessing the evolution of the grain size distribution of unbound granular materials is extremely complex because particle breakage involves several characteristics of the mixture such as grain angularity, grain size distribution, particle strength, porosity, stress level, anisotropy, and water content [1–9].

Researches carried out in the past 15 years proved the capability of discrete element models, DEM, to analyse crushing of granular materials [5,10–15]. These models permit the calculation of loads between grains and crushing of individual particles; however, such models require adopting restrictive assumptions concerning the number of particles involved in the model and the shape of each particle. Unfortunately, a better approach to the actual state of a granular material using DEM requires a high computational cost. For this reason, the development of analytical models requiring low computational costs is necessary to include crushing in numerical models based on continuum mechanics [16,17]. The model proposed in this paper includes a linear packing model to describe polydisperse mixtures and Markovian processes to assess the evolution of the grain size distribution of granular mixtures under cyclic loading.

The comparison of the results obtained experimentally with those obtained using the proposed model shows very good agreement. For this reason, the analytical model proposed here could be a useful alternative to be included in models based in continuum mechanics.

The paper is divided in two parts: part 1 includes the theoretical background of the model, its relevance to describe granular compaction and the experimental validation of the model input parameters and of granular compaction results; part 2 describes the model for comminution of granular materials under cyclic loading including the theoretical model development and its validation.

2. Part 1: Description of a granular material as a polydisperse mixture using a linear packing model, LPM

The description of the evolution of the grain size distribution of a granular material requires a model providing some characteristics of a continuum material from a mixture of grains of different size. Models dealing with binary mixtures of clay and granular particles have been proposed by several authors [18–20]; on the other hand, models working with polydisperse mixtures have been proposed for concrete technology to find out mixtures of granular materials minimizing void spaces. The linear packing model used

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Nomenclature

List of symbols

a_{ij}	decompaction coefficient for the linear packing model
b_c	size coefficient for the Weibull function
b_f	slope of the Wohler fatigue line
b_{ji}	wall effect coefficient for the linear packing model
Cu	coefficient of uniformity of the grain size distribution
d_i	size of particles i
e	void ratio
F_c	filling coefficient $F_c = \Phi_i/\Phi_i^*$
F_c^c	complement of the filling coefficient $F_c^c = 1 - F_c$
f_i	contact force between particles i
K	packing coefficient
k	coefficient for the probability distribution function of contact loads
K_i	packing coefficient of grains i
m	coefficient of the Weibull function
n	porosity
n_c	number of classes in the grain size distribution
N	number of loading cycles
P_{ij}	transition probability for the Markov transition matrix
$P_f(d_i)$	failure probability of grains i
$P_s(d_i)$	survival probability of grains i
y_i	volumetric proportion of grains i
α	coefficient for the probability distribution function of contact loads
α_β	coefficient of the Beta function representing the distribution of crushed grains
α_c	exponent of the function affecting crushing depending on the compacity of the mixture
β	coefficient for the probability distribution function of contact loads
β_i	residual compacity: maximum compacity obtained experimentally for the fraction i individually
β_β	coefficient of the Beta function representing the distribution of crushed grains
χ_f	proportion of floating particles in the mixture
Φ	compacity: volume of grains by unit volume
Φ_i	partial volume of the class i: volume of the class i by unit volume
Φ_i^*	maximum partial volume of grains i considering the presence of other grains in the mixture
$(\Phi_i/\Phi_i^*)_{mc}$	filling coefficient for maximum comminution
γ	virtual compacity: theoretical maximum compacity of the granular mixture without any alteration of the particle's shape
γ_i	virtual compacity considering grains i as dominant
σ_0	characteristic strength
σ_M	macroscopic strength
$\langle\sigma\rangle$	mean stress between particles
$\langle\sigma\rangle_N$	normalized mean stress between particles
ξ	normalized contact load between particles

in this paper is based on the model for polydisperse mixtures proposed in [21–23].

2.1. Definitions for the linear packing model

The linear packing density model allows calculating the compacity of polydisperse granular mixtures [21]. The derivation of the equations that allows calculating the compacity of the granular mixture requires the definitions given in Table 1.

The compacity, Φ, is defined as the solids volume of grains within the mixture by unit volume, it is directly related with the volume of voids in a granular material given by the porosity n, or the void ratio e as follows:

$$n = 1 - \Phi, \quad e = \frac{1}{\Phi} - 1 \tag{1}$$

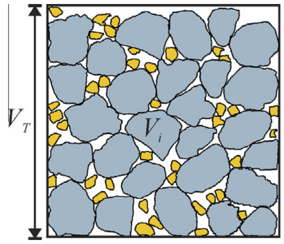
The linear packing density model permits obtaining the compacity of a granular mixture, Φ, knowing the volumetric proportion, y_i, of particles of size d_i. This calculation is based on the evaluation of the virtual compacity, γ, defined as the maximum compacity theoretically reachable in the granular mixture. This evaluation requires the knowledge of the residual compacity β_i of each grain size, which represent the maximum compacity obtained experimentally for each fraction individually. As the virtual compacity γ results from a calculation based on β_i, the particle's shape of each fraction within the mixture correspond to the shape of fraction i when measuring β_i.

2.2. Virtual compacity of granular mixtures

For binary mixtures with grain sizes d_1 and d_2 three cases could be considered depending upon the interaction between the two classes of grains [22,23]:

- No interaction: in this case one of the grain class is substantially bigger than the other, $d_1 \gg d_2$, as a result the local structure of each type of grains is unaltered by the presence of the other grains. Two cases are possible, Fig. 1a shows the case the mixture is supported by the arrangement of big grains and the small grains are placed within the voids; although in Fig. 1b the mixture is supported on the smaller grains and the big grains are immersed in the space of small particles.
- Total interaction occurs when the size of the particles in the binary mixture is identical, but the residual compacity is different: $d_1 = d_2$; $\beta_1 \neq \beta_2$, Fig. 2a.
- Partial interaction occurs in the case of binary mixtures with $d_1 > d_2$. In this case, two physical effects appear: decompaction effect created by the small grains filling voids, Fig. 3a; and boundary effect appearing in the contact between small and big grains, Fig. 3b.

Table 1
Definitions for calculating the virtual compacity.



n_c: number of grain classes
V_i: volume of grains of the class i
V_T: total volume
V_V: voids volume
V_S = solids volume, $V_S = \sum_{i=1}^{n_c} V_i$
n: porosity, $n = V_V/V_T$
e: void ratio, $e = V_V/V_S$
β_i: residual compacity, $\beta_i = \max(V_i)/V_T$[a]
Φ_i: partial volume of class i, $\Phi_i = V_i/V_T$
Φ: compacity of the mixture, $\Phi = V_S/V_T$, $\Phi = \sum_{i=1}^{n_c} \Phi_i$
y_i: Volumetric proportion of grains i, $y_i = \Phi_i/\Phi$
γ: virtual compacity, $\gamma = \min(\gamma_i)$[b]

[a] The maximum volume of fraction i, $\max(V_i)$, is obtained taking each fraction separately and compacting it to its maximum density as described in Section 3.2.1.
[b] γ_i is obtained using Eq. (10).

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