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Research Paper

Slope stability analysis by polar slice method in rotational failure mechanism

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ABSTRACT

In this paper, a polar slice method is proposed to quantify the energy dissipation in the inner sliding soil mass and evaluate the slope stability in rotational failure mechanism. The polar slice method is compared with the vertical slice method. Despite similar results, the advantages of the polar slice method are explained. Besides, the comparison between the polar slice method and Chen's method shows that the polar slice method would lead to more accurate results than Chen's method, while Chen's method which neglects the energy dissipation in the inner sliding soil mass is relatively conservative.

1. Introduction

Slope stability is always a research focus in geotechnical engineering. Many methods are developed to evaluate slope stability, such as limit equilibrium method (LEM) [1,2], finite element method (FEM) [3] and limit analysis (LA) [4].

Limit analysis was popularized by fundamental theory and practical applications by Chen [5–7], and it has been employed for evaluating the slope stability by many researchers. The rotational failure mechanism for slopes was considered by Chen [5]. It is the most adverse known failure mechanism in slopes of homogeneous soil, and it was found to be the case also for reinforced soil slopes by Michalowski [8]. Kinematic admissibility requires the sliding surface of slope be a log-spiral in Chen's method, and it has been adopted in slope analysis by many researchers [9–13]. Chen's method assumes that the sliding soil mass is one rigid body so the energy dissipation in the inner sliding soil mass is not considered. However, according to the geometry of log-spiral, the polar radius increases as polar angle increases, so the sliding soil mass has to deform to adapt its shape to different curvature, which is against the assumption of one rigid body in Chen's method.

To solve this problem, Michalowski [14] presented a vertical slice method to divide the sliding soil mass into several small blocks so that the failure lines at the slice bottom are estimated with plane surfaces which is similar to Janbu and simplified Bishop Methods. This method transforms the complex sliding movement into translational movement along all the plane surfaces, and the energy dissipation in the inner sliding soil mass is considered by calculating the energy dissipation along the vertical interfaces. Michalowski [14] also applied the vertical slice method into rotational failure mechanism and compared the results with Chen's method. Even so, the vertical slice method provides a complex application in rotational failure mechanism. To improve this, a polar slice method is proposed in this paper. Besides, the polar slice method is compared with both Chen's method and the vertical slice method. The advantages of the polar slice method are explained, and it would provide reference for slope stability analysis in rotational failure mechanism.

2. Comparisons between Chen's method and vertical slice method

Chen's method and vertical slice method are illustrated in Fig. 1 respectively. The failures are both assumed to obey the Mohr-Coulomb criterion and associative flow rule. The shear strength reduction (SSR) technique used in LEM has been adopted in many papers [15–17], and it is also employed in this paper to obtain the safety factor F_s under static conditions, so the mobilized strength parameters c_m and φ_m can be expressed below in Eq. (1) [18,19].

$$\left.\begin{aligned} c_m &= c/F_s \\ \tan\varphi_m &= \tan\varphi/F_s \end{aligned}\right\} \quad (1)$$

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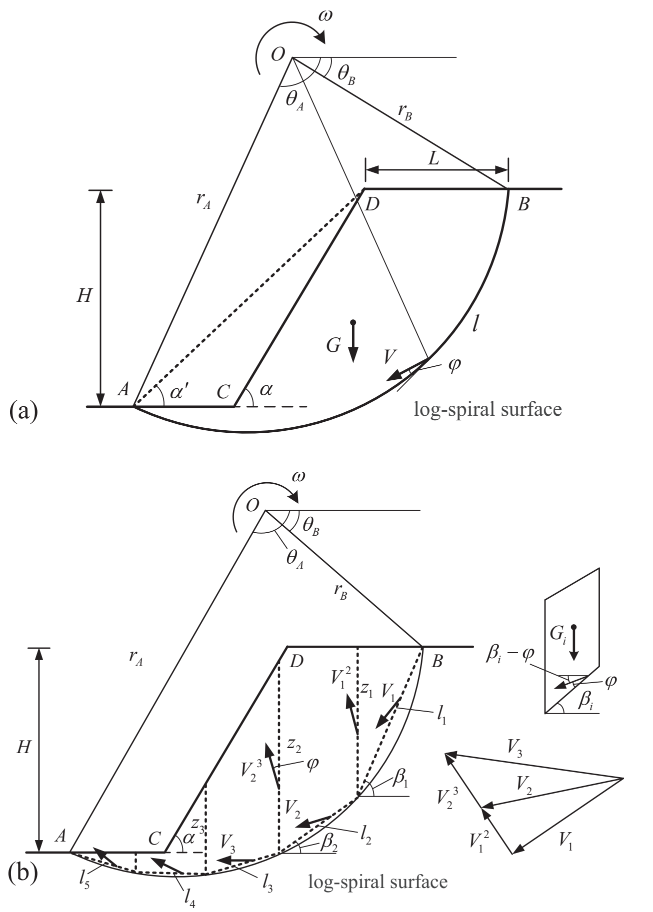

Fig. 1. Two methods in rotational failure mechanism: (a) Chen's method. (b) Vertical slice method.

According to Chen's method, the rate of external work due to body forces and tractions is caused by the weight of the soil, G in Fig. 1(a), and energy dissipation only occurs along the log-spiral surface. So the energy-work balance equation at the limit state is listed in Eq. (2).

$$\left.\begin{array}{l} Q_c = Q_G \\ Q_c = \int_{\theta_B}^{\theta_A}(cV\cos\varphi)\frac{r\,\mathrm{d}\theta}{\cos\varphi} = cr_B^2\omega f_c \\ Q_G = \gamma r_B^3\omega(f_1 - f_2 - f_3 - f_4) \end{array}\right\} \quad (2)$$

where Q_c is the rate of energy dissipation along the log-spiral surface, Q_G is the rate of work due to gravity, c and φ are cohesion and internal frictional angle respectively, V is the velocity along the sliding surface, r_B is the polar radius which is shown in Fig. 1 (a), γ is unit weight of soil, ω is angular velocity of the sliding soil mass. f_c, $f_1 - f_4$ are intermediate variables which are related to the failure mechanism and geometry of the slope, hence they are functions of angles θ_A, θ_B, α', α, φ. Detailed expressions of these variables can be obtained by referring to Ref. [4].

As for the vertical slice method proposed by Michalowski [14], the rate of work due to gravity is to sum up the rate of work of all slice weight, and the energy dissipation is to sum up the energy dissipation on all slice bottom planes and the vertical interfaces. Meanwhile, the shear strength parameters on the vertical interfaces are assumed to be equal to the ones on the slice bottom planes. So the energy-work balance equation of vertical slice method is listed in Eq. (3).

$$\left.\begin{array}{l} Q_l + Q_z = Q_G' \\ Q_l = \sum_{i=1}^{n} cl_iV_i\cos\varphi \\ Q_z = \sum_{i=1}^{n-1} cz_iV_i^{i+1}\cos\varphi \\ Q_G' = \sum_{i=1}^{n} G_iV_i\sin(\beta_i - \varphi) \end{array}\right\} \quad (3)$$

where Q_l is summation of the energy dissipation along the slice bottom planes, Q_z is summation of the energy dissipation along the vertical interfaces, which is a quantitative method for the energy dissipation in the inner sliding soil mass. Q_G' is summation of the rate of work due to slice weight. n is the number of the slices, l_i is the length of the slice bottom planes, and z_i is the length of the vertical interfaces between the slices, V_i is the velocity at the slice bottom planes, V_i^{i+1} is the relative velocity on the vertical interfaces, β_i is the inclination angle of the slice bottom planes.

It's concluded that Q_G' and Q_l in Eq. (3) are equivalent to Q_G and Q_c in Eq. (2) respectively, so the difference between Eq. (2) and Eq. (3) is concerning Q_z. Chen's method assumes the sliding soil mass as one rigid body so the energy dissipation in the inner sliding soil mass is not considered. While in the vertical slice method, the energy dissipation occurs not only along the slice bottom planes, but also within the sliding soil mass, along the vertical interfaces. The energy dissipation in the inner sliding soil mass is quantified by Q_z.

It's mentioned above that it is irrational to assume the sliding soil mass to be one rigid body, so the energy dissipation in the inner sliding soil mass should be included in the energy equation. The vertical slice method provides a general way to calculate this for sliding surface of arbitrary shape. But in the rotational failure mechanism, the main variables are presented in polar coordinates, while the vertical slice method cannot employ the variables directly, and the variables have to be transformed from the polar coordinates into rectangular coordinates which would lead to complex application. Therefore, a polar slice method is proposed for the rotational failure mechanism in this paper. In this method, energy dissipation in the inner sliding soil mass would be obtained by the summation of the energy dissipation on all the polar interfaces. Other work rate would be calculated the same as Chen's method because Q_c and Q_G in Eq. (2) are both analytic expressions which are more precise than the summation expressions of Q_l and Q_G' in Eq. (3).

3. The principle of polar slice method

Generally, the energy dissipation in the inner sliding soil mass results from inner friction and it is not likely to be directly computed. Slice method provides an indirect method to estimate the energy dissipation in the inner sliding soil mass. The schematic diagram of the principle of polar slice method is shown in Fig. 2. The polar angle range $\theta_A - \theta_B$ is isometrically partitioned into N parts to generate $N + 1$ block velocities on the sliding surface which is shown in Fig. 2(a). According to the theory of admissible velocity fields, an interface of relative velocity would be generated between each pair of adjacent block velocities, which means N interfaces would be generated. For consistency of calculation, it's assumed that the extension lines of all interfaces pass through the pole of log-spiral. The relative velocity V_i^{i+1} would make an angle φ with the interface, and the adjacent block velocities V_i, V_{i+1} on the sliding surface will form a closure triangle together with the relative velocity V_i^{i+1} on the interface which has been shown in Fig. 2(b).

In order to analyze the relationship between the three velocity vectors, the azimuth angle ψ is defined from x axis in the anti-clockwise direction to the vectors. So the azimuth angle ψ_i, ψ_{i+1} can be expressed below: