Computers and Geotechnics 80 (2016) 41–48

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Three-dimensional topology optimization for geotechnical foundations in granular soil

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ARTICLE INFO

Article history:
Received 15 October 2015
Received in revised form 20 June 2016
Accepted 21 June 2016

Keywords:
Structural optimization
Hypoplasticity
Finite element modelling
Three-dimensional

ABSTRACT

This article presents three-dimensional structural optimization in geotechnical engineering for foundations in granular soil. The general design (topology) of a shallow foundation is optimized with respect to its deformational behaviour within the service limit state. The SIMP (solid isotropic material with penalization) method is applied to optimize the distribution of foundation material. The soil is modelled as a hypoplastic material with a constitutive model suitable for optimization using finite element analysis. Two load cases are examined. The optimized topology is validated against two-dimensional optimization and 1g-model test results. The present study proves the applicability and shows the potential of topology optimization in geotechnical engineering.

1. Introduction

Geotechnical design usually consists of common structural elements such as piles, shallow foundations, walls or anchors. Those are combined to a system that fulfils stability and serviceability requirements. Contrary to other engineering fields, like automotive or aircraft engineering, numerical structural optimization methods are scarcely used in the geotechnical design process. However, by applying these methods the structural design may be adapted optimally to problem specific demands, which saves expenses, economizes material use, improves the deformational behaviour and reduces the construction time.

Within the structural optimization three branches may be distinguished: dimension optimization, shape optimization and topology optimization.

Out of these the dimension optimization is the best known and often applied in geotechnical engineering. It comprises the optimization of construction parts through e.g. the variation of material thicknesses, element lengths or diameters. Some examples are the sizing of a shallow foundation [1,2], of a pile group [3] or grillage [4], of the reinforcement of a concrete pile [5] and of a quay wall [6].

A broader spectrum of the possible design results from shape optimization since angles and form of the construction or several construction parts may be varied during optimization. This type of structural optimization has, among others, already been applied to piled raft foundations [e.g. 7], to pile grillages [e.g. 8,2], to quay wall constructions [e.g. 6] and underground excavations [e.g. 9–11].

Using topology optimization it is possible to generate an optimized structure from an unshaped block. No preset solutions have to be provided[1] and therefore the possible optimization result is not restricted to commonly known structural elements. The basic structural set-up, the so called topology, can be determined efficiently based only on the knowledge of loads and limiting boundaries. Therefore topology optimization is able to support the design process at early stage. However, the herein presented application of topology optimization to geotechnics has only recently been initiated in research. The topology optimization of an underground excavation in linear-elastic material is shown in [13–17,10] and in nonlinear material is shown in [18,12,19]. The topology optimization of a foundation beneath a strip footing in granular Mohr-Coulomb material [2] and in granular hypoplastic material [20–23] has been presented.

So far the application of topology optimization in three dimensional problems in geomechanics has been limited to finding the best shape of underground openings [13,11]. This paper presents the application of topology optimization to a three-dimensional

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[1] In special cases due to construction and other restrictions it might be preferable to limit the feasible design space. For example see the ground structure approach used in [12].

http://dx.doi.org/10.1016/j.compgeo.2016.06.012
0266-352X/

geotechnical foundation problem to examine whether it yields plausible and efficient solutions, also in more realistic three-dimensional modelling. The three-dimensional model is based on a two-dimensional model and corresponding 1g-model tests in [21,22], which are used to validate the three-dimensional results against.

2. Numerical method

Structural optimization usually comprises the coupling of mathematical optimization methods with numerical simulation, e.g. the Finite Element Method (FEM). The following sections describe the numerical method arising from this coupling.

2.1. Topology optimization with SIMP 3D

The numerical structural optimization can be conducted using a density-based approach to topology optimization as proposed by Bendsøe [24] and Zhou and Rozvany [25].[2] Using this approach the topology optimization problem is considered as a material distribution problem in which the relative material distribution is parameterized. A design domain is discretized using finite elements. The optimization variables are relative material densities ρ which are assigned to the finite elements. The material properties of each element e, e.g. its stiffness are linked to its relative material density ρ_e, which may take on values between 0 and 1. However, in order to obtain structural design with crisp boundaries, intermediate values for the relative material density have to be excluded. Therefore a power-law approach is applied, which is referred to as **S**olid **I**sotropic **M**aterial with **P**enalization (SIMP). For the conducted analysis the element's stiffness E_e is defined by the modified SIMP-approach e.g. [27]:

$$E_e(\rho_e) = E_{\text{soil}} + \rho_e^p (E_{\text{struc}} - E_{\text{soil}}), \quad \rho_e \in [0,1] \tag{1}$$

Considering the topology optimization of a geotechnical structure E_{soil} represents the stiffness of the surrounding subsoil and E_{struc} the stiffness of the structural material. p is the penalization factor. In order to prevent the convergence to local minima the penalization factor is continuously increased. Adapting a continuation formulation [28], the penalization factor p using the iteration number k is given by

$$p^k = \begin{cases} 1 & k \leqslant 10, \\ \min[p_{\max}, 1.021 p^{k-1}] & k > 10, \end{cases} \tag{2}$$

where $p_{\max} = 5$. The chosen formulation results in lower system's strain energy after 100 iterations compared to the original stepwise increase of the penalty factor p, Fig. 1, and will, therefore, be used in all presented examples.

Several computational codes for topology optimization can be found in literature, of which the efficient three-dimensional code from Liu and Tovar [30] is chosen as basis for the presented application. The code is written in Matlab and proceeds earlier two-dimensional codes [31,32]. The three dimensional code has an inbuilt FE-analysis, which has been modified to link with Abaqus/Standard instead to meet the complexity requirements of the analyzed geotechnical problem. Additional necessary adaptations for geotechnical topology optimization are described in the following.

The minimum compliance problem is adapted in the presented application. The objective is to find a material distribution within the design domain that minimizes the system's strain energy which implicitly minimizes the deformations. Pucker [21] proposes to evaluate the strain energy at the integration points. The structure's strain energy c is therefore defined as a sum over all

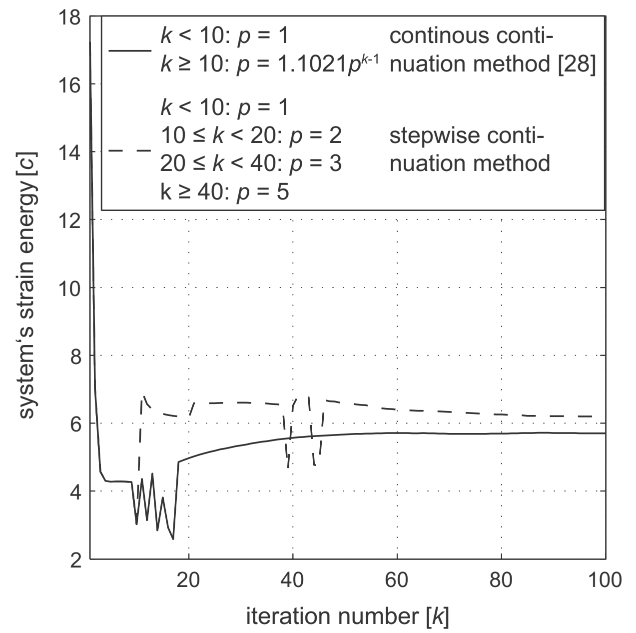

Fig. 1. Development of system's strain energy during optimization for the excentric model and $vf = 0.46875\%$ for two continuation methods for the increase of the penalty factor p according to the iteration number k. A gradually increasing continuation method (Eq. (2)) modified after [28] marked with the solid line and a stepwise increasing continuation method [e.g. 29].

integration points g of the FE-model [21]. The optimization problem (Eq. (3)) is subject to a volume constraint and the constrained domain for the relative material density ρ_e:

$$\begin{aligned} \min: \; & c(\rho) = \varepsilon^T \mathbf{C} \varepsilon = \sum_e^n \sum_g^m (\rho_{e,g})^p \varepsilon_{e,g}^T \mathbf{C}_{e,g} \varepsilon_{e,g} \\ \text{subj. to}: \; & \frac{V(\rho)}{V_0} = vf \\ & 0 < \rho_{\min} \leqslant \rho_e \leqslant 1, \end{aligned} \tag{3}$$

where ε denotes the strain tensor, $\mathbf{C}$ denotes the material tensor, V denotes the material volume and vf the volume fraction within the design domain.

The optimality criteria (OC) method is used for optimization. This method is derived from the Karush-Kuhn-Tucker conditions for optimality. A basic updating scheme based on the OC-method is presented by Bendsøe and Sigmund [33]. It has been modified to include a gray-scale filtering technique by Groenwold and Etman [34]. The herein presented optimization uses Groenwold and Etman's formulation [34]:

$$\rho_e^{\text{new}} = \begin{cases} \max(\rho_{\min}, \rho_e - m) & \text{if} \quad \rho_e B_e^\eta \leqslant \max(\rho_{\min}, \rho_e - m), \\ \min(1, \rho_e + m) & \text{if} \quad \min(1, \rho_e + m) \leqslant \rho_e B_e^\eta \\ (\rho_e B_e^\eta)^q & \text{otherwise}, \end{cases} \tag{4}$$

where $m = 0.2$ is a moving limit, which constraints the change for the updated relative density, $\eta = 0.5$ is a numerical damping coefficient and q is a variable for gray scale filtering [34]. According to the optimality condition B_e is given as

$$B_e = \frac{-\frac{\partial c}{\partial \rho_e}}{\lambda \frac{\partial V}{\partial \rho_e}}, \tag{5}$$

where λ is the Lagrangian multiplier for volume constraint. The derivation of strain energy (also referred to as sensitivity) for the modified SIMP approach is adapted from [21] and given by

[2] For further approaches to topology optimization see e.g. [26].